occasional dropping of the dot in the scalar product of two vectors, and so on, there may be found logical errors which may be corrected by changing a "the" to an "a" or to "certain," or "both" to "the two" (pp. 53, 55), and errors such as "a virtual displacement is said to be every displacement of the point A . . . " (pp. 470, 471). Such slips are, however, not numerous enough to be very serious. An error of another kind arises from the inclusion of a factor 1/6 in a formula for the volume of a parallelepiped (p. 13).

The most serious blunder which the reviewer noted is the statement (p. 144) that if the density of the earth is distributed symmetrically with respect to the center of mass then it can be proved that the force of attraction is directed constantly toward the earth's center of mass. It is a vitally important fact in celestial mechanics that this is not so; the conclusion is incorrect even for a homogeneous oblate spheroid. It is an interesting problem to determine for what surfaces and what laws of density the attraction of a body for a particle exterior to it is constantly directed toward the center of mass of the body. The appropriate vector equation, equivalent to three scalar equations, is an integral equation for the density function, has an unknown function in an equation of the surface of the body, and a third unknown proportionality function. Solutions to this problem are known, but they do not agree with the one in the book.

We must express our appreciation to the author, to the translator, and to the publisher for adding this fine book to the collection of works on classical mechanics.

E. J. MOULTON

The theory of the Riemann zeta-function. By E. C. Titchmarsh. Oxford University Press, 1951. 6+346 pp. $8.00.

The zeta-function was introduced almost 100 years ago by Riemann in his famous memoir on the number of primes less than a given number. While since then enough has been discovered about the zeta-function to justify its use in analytic number theory, the question raised by Riemann about the location of its zeros remains unanswered. The milder hypothesis of Lindelöf that $\zeta(1/2+it)=O(t^{\epsilon})$ for every $\epsilon>0$ also remains unsettled. The zeta-function continues to be a major challenge to mathematicians.

The author in his well known Cambridge Tract of 1930 gave a remarkably comprehensive and concise account of the zeta-function. Now he has given an expanded account in order to include recent results of which the most notable are due to A. Selberg. The sole prerequisite for reading this treatise is a knowledge of the fundamentals of function theory. The author is an excellent expositor of the kind of analysis that up to now has been the major tool in research on the zeta-function.

A sketch of the contents of the book follows. The first chapter gives the Euler product and Dirichlet series representation of $\zeta(s)$. Representation of $\log \zeta(s)$, $1/\zeta(s)$, $\zeta^k(s)$, etc., are also given as well as series involving Ramanujan sums as coefficients.

The second chapter deals with the analytic character of $\zeta(s)$. It contains seven methods for proving the functional equation.

The third chapter contains the Hadamard-de la Vallée Poussin theorem that $\zeta(1+it)\neq 0$ and the prime number theorem, and considers $\zeta(s)$ and related functions for s slightly to the left of the line $R(s)=1$.

The fourth chapter takes up approximate formulae for $\zeta(s)$ including the very useful approximate functional equation.

The fifth chapter is on the order of $\zeta(s)$ in the critical strip and uses the methods of Weyl as developed by Hardy and Littlewood and of van der Corput for the estimation of exponential sums which occur in the approximate formulae for $\zeta(s)$. In the sixth chapter exponential sums are treated by Vinogradoff's method.

The seventh chapter treats the mean values of powers of $|\zeta(s)|$ along lines $R(s)=$constant in the critical strip. The approximate formulae for $\zeta(s)$ are used as is the convexity of the mean values of analytic functions.

The symbol Ω is used to denote negation of o so that $F(t)=\Omega(\phi(t))$ means that there exists an $A>0$ such that $|F(t)|>A\phi(t)$ for some arbitrarily large values of t. In chapter eight Ω theorems for $\zeta(\sigma+it)$ and $1/\zeta(\sigma+it)$ are obtained by diophantine methods and also by dealing directly with integrals of high powers of the functions being studied.

The distribution of zeros in the critical strip is considered in chapter nine and recent results of A. Selberg are included. The zeros on the critical line are studied in chapter ten. Many proofs are given of Hardy's theorem that there are an infinite number of zeros on the critical line. The number of zeros of $\zeta(s)$ of the form $1/2+it$, $0<t\leqq T$, is denoted by $N_0(T)$. The result of A. Selberg that $N_0(T)>AT\log T$ for some $A>0$ and all large T is given. (It will be recalled that if the Riemann hypothesis is true $N_0(T)\sim(1/2\pi)T\log T$.)

In chapter eleven the general distribution of values of $\zeta(s)$ is studied. Near the line $R(s)=1$, Picard's theorem is used. In the critical strip diophantine approximation is used.

Let the number of divisors of n be denoted by $d(n)$ and $\sum_{n\leqq x} d(n)$

$=x \log x+(2\gamma-1)x+\Delta(x)$. It was shown by Dirichlet that $\Delta(x) = O(x^{1/2})$ and better results have been obtained since. The function $\Delta(x)$ is related by the Mellin transform to $\zeta^2(w)/w$ in the critical strip. The divisor problem, that is, the order of $\Delta(x)$, and generalizations are considered in chapter twelve.

In chapter thirteen the Lindelöf hypothesis is assumed to be true and consequences of it are investigated and in chapter fourteen consequences of assuming the truth of the Riemann hypothesis are investigated.

The final chapter is on calculations relating to the zeros of $\zeta(s)$. After indicating how the early zeros are shown to lie on the critical line the author observes that if the Riemann hypothesis is false this could be shown by using modern calculating devices.

N. LEVINSON

Advances in applied mechanics. Vol. II. Ed. by R. von Mises and T. von Kármán. New York, Academic Press, 1951. 10+233 pp. $6.50.

Kármán and Lin's paper, *On the statistical theory of isotropic turbulence*, represents the views of two distinguished specialists in the current semi-empirical turbulence theory. It purposes to clarify the notion of spectral similarity in flows where there is turbulent diffusion of energy. The authors divide the spectrum into three frequency ranges, in each of which only two parameters are considered significant; similarity laws follow by the usual dimensional argument. Since this paper is of a nonmathematical character, the reviewer does not detail its contents here.

Kuerti's survey, *The laminar boundary layer in compressible flow*, is successful in its expressed aim: "Although the mathematics used in the boundary layer literature presents no particular conceptual difficulties, it is complicated on account of the number of parameters involved and because of the different approaches tried by different authors. The study of the original literature thus requires a patient reader. Under these circumstances it seemed desirable to put together what may be called a guide to rather than a review of the existing literature on the subject." Attention is restricted to steady plane flow of a perfect gas with constant specific heats, the viscosity and thermal conduction being assumed functions of temperature only, and the Prandtl number constant. The bibliography of thirty items seems to be complete through 1948.

The author does not mention any work of his own, and there appears to be no original contribution made by the article. One notes, for example, that the derivation in Part II is based upon the usual