In the latter part of the book, the general recursive functions themselves (in the absolute sense) and their explicit form are treated very fully, but the associated notions and applications are treated less fully or only cited. In writing this book Mrs. Péter has carried out a considerable undertaking; and to go further would have constituted a still greater one, and required either a much larger book or a more compact style.

Only a minimum of knowledge of elementary number theory, analysis, and set theory including transfinite ordinals is presupposed and none of mathematical logic. Mrs. Péter aims to make the subject intelligible to the beginner by working out the treatment of many topics (particularly in the special theory) on an example, whence the reader can surmise how the treatment would go in general (or consult the literature). This method has both advantages and disadvantages. No student can complain that he has lost contact with the reality for want of concrete examples; but an unwary reader may be oppressed by the immense amount of detail involved in working out the examples and proofs.

S. C. KLEENE

Infinite matrices and sequence spaces. By R. G. Cooke. London, Macmillan, 1950. 14+347 pp. 42s.

This book, which might be considered a continuation of Chapter XII of Dienes' *Taylor series*, is a useful and welcome adjunct to the recent book by Hardy, *Divergent series*, Oxford, 1949. The overlap between these is slight since the present book is largely concerned with the study of general properties of classes of regular summability transformations.

Chapter 1 introduces several special classes of infinite matrices and certain of the special problems that arise in connection with their algebraic properties such as the ever-present need for the consideration of the validity of interchange of limit operations which leads, for example, to the failure of the associative law. Chapters 2 and 3 deal with the existence of left- and right-hand inverses and annihilators in rings of matrices, the notion of a "bound" (norm) and of weak convergence of sequences of matrices, and the special problem of solving the equations $AX=XD$ and $AX-XA=I$ for specified A and diagonal D. In Chapter 4, the class of K-matrices which transform convergent sequences into convergent sequences is characterized, as well as the subclass of Toeplitz T-matrices and the analogous classes of series to sequence transformations. The Steinhaus theorem and numerous variants are proved, together with others showing what may be achieved by a general T-matrix. Chapter 5 deals with the general question of consistency of two matrices A and B. Numerous results are obtained in the case of commutative A and B. The chapter concludes with a discussion of the relation between a matrix and its left or right translate, and between the iterative product and composition product of two matrices, as developed by Agnew. Chapter 6 is the best in the book and gives a very complete treatment of the behavior of the core of a sequence under T transformations. In particular, several new proofs and theorems due to A. Robinson are included. Chapters 7 and 8 discuss the efficiency of classes of T matrices as applied to the summability of power series, to bounded sequences, and to sequences of 0's and 1's. Chapter 9 contains a detailed account of the elementary theory of the Hilbert space l^2 and to the theory of bounded bilinear forms $A(x, y)$, similar in treatment to that contained in Hardy, Littlewood, and Pólya, *Inequalities*, Cambridge, 1934. In Chapter 10, certain linear spaces of sequences are discussed, together with their duals, in the spirit of Köthe-Toeplitz; various types of weak convergence are defined and their relations investigated, using results of H. S. Allen.

The book is clearly written with a profusion of details, and the bibliography is extensive, containing some papers as recent as 1950. The author is especially to be commended for the excellent sets of problems which follow each chapter. It is to be regretted that the book as a whole is so remote in point of view and method of treatment from the current trend of research in the fields most closely allied with the material presented. This extends not only to terminology (for example, the author's use of "field" to describe a possibly non-associative ring of matrices) but also to the entire approach. Chapters 2, 3, 4, 5, 9, and 10 would have been much improved if presented from the point of view of convex linear topological spaces, topological algebras, and their accustomed machinery of dual spaces of functionals, weak and strong topologies, and operators. It is noteworthy, for example, that the discussion of Hilbert space contains no mention of operators. The final chapter does contain a brief description of a Banach space and a proof of the Baire category theorem, and Chapter 3 contains a cursory discussion of the notion of a group algebra, but these are not germane to the rest of the book, and their full power is not used. Along more classical lines, the reviewer finds the author's remarks on the "right" value for the sum of a divergent series in

§8.7 (which is defined to be the Abel limit) extremely disturbing, especially in the light of the well known fact that Abel and Cesaro summability coincide for bounded sequences.

R. CREIGHTON BUCK

Randwertprobleme und andere Anwendungsgebiete der höheren Analysis für Physiker, Mathematiker und Ingenieure. By F. Schwank. Leipzig, Teubner, 1951. 6+406 pp. $5.47.

The book is a compendium of those portions of mathematical analysis beyond advanced calculus which are of great interest to engineers and physicists. It is an excellent book for physicists and engineers in that it provides a lucid introduction to a good selection of mathematical techniques and theories, with references for further reading, examples, practical applications, and an almost encyclopedic bibliography of applications. From the point of view of the mathematical reader the principal merit of Schwank's book is the wide range and amazingly large number of practical applications described or quoted in the various chapters.

In a preface, G. Hamel explains that Schwank's book is neither a textbook, nor a work of reference. Although the presentation is consequential and covers the ground thoroughly, the book is not as systematic as a textbook, and not as complete as a book of reference. It is written for the physicist or engineer with only a modest mathematical knowledge, and a desire to learn more about some of the more advanced mathematical techniques. The author aims at precision wherever it can be attained. In view of the readers for whom the book was written, it is quite clear that precision and rigour could not be maintained throughout the book, and examples and descriptions are called in when general formulations and proofs would seem out of place.

The material is organized in six chapters and a mathematical appendix.

Chapter I is an introduction to boundary value problems. The vibrations of a string are discussed in detail, and boundary conditions, normal modes of vibration, characteristic values, characteristic functions, orthogonality, Fourier expansion, and other relevant notions are introduced. D'Alembert's solution of the one-dimensional wave equation is also given. The last two sections of this chapter are devoted to the re-formulation of the problem of the vibrating string in terms of integral equations and calculus of variations respectively.

Chapter II is devoted to complex variables. It starts from the