The volume concludes with two papers: Paper no. 14, by Dresher, Karlin, and Shapley, is concerned principally with zero-sum two-person games with the kernel $K(x, y) = \sum_{i,j} a_{ij}r_i(x)s_j(y)$, where the functions $r_i(x)$ and $s_j(y)$ are continuous, and x and y, the pure strategies of the two players, are points on the unit interval. Paper no. 15 by Bohnenblust, Karlin, and Shapley, discusses zero-sum two-person games with payoff function $M(x, y)$, which is such that, for every x in an arbitrary set A, $M(x, y)$ is a continuous convex function of y. The y-player is shown to have an optimal pure strategy (y is a point in a compact, convex n-dimensional region of a finite Euclidean space). The main result of the paper is that the x-player has an optimal mixed strategy which assigns probability one to a set which contains at most $(n+1)$ points x.

This volume has the merit of bringing together between two covers a large number of interesting papers on game theory which would otherwise be inconveniently scattered over many periodical issues. Such enterprises could profitably become a feature also in other branches of mathematics. The editors deserve much credit for a painstaking job, and for their lucid and stimulating introduction.

J. WOLFOWITZ

Hydrodynamics, a study in logic, fact, and similitude. By Garrett Birkhoff. Princeton University Press, 1950. 14+186 pp. $3.50.

The material of this book formed the contents of a series of lectures at the University of Cincinnati in 1947, and this perhaps accounts for the unusual and unconventional choice of material for a book with the title *Hydrodynamics*. There are five chapters in the book, most of which have only a loose connection with the others.

The reviewer found it difficult to understand for what class of readers the first chapter was written. For readers who are acquainted with hydrodynamics the majority of the cases cited as paradoxes belong either in the category of mistakes long since rectified, or in the category of discrepancies between theory and experiment the reasons for which are also well understood. On the other hand, the uninitiated would be very likely to get wrong ideas about some of the important and useful achievements in hydrodynamics from reading this chapter. In the case of air foil theory, for example, the author treats only the negative aspects of the theory. It has always seemed to the reviewer that the Kutta-Joukowsky theory of airfoils is one of the most beautiful and striking accomplishments in applied mathematics. The fact that the introduction of a sharp trailing edge makes possible a physical argument, based on consideration of the effect of viscosity, that leads to a purely mathematical assumption regarding the behavior of an analytic function which in its turn makes the solution to the flow problem unique and also at the same time furnishes a value for the lift force, represents a real triumph of mathematical ingenuity. The fact that the theory does not solve all of the problems concerned with airfoils should not be surprising; no physical theory is every final nor without limits within which it is valid. These aspects of the Kutta-Joukowsky theory are not mentioned by the author.

At the end of the first chapter some general observations regarding the philosophy and correct attitude toward applied mathematics are made and some questions are raised. Most workers in the field would agree quite well with the author's observations, though they are perhaps better informed in some cases than the author would seem to imply. For example, they know quite well why it is that a small change in viscosity may have radical consequences while a small change in compressibility has little effect. The reason for this is clear from the mathematical formulation: the small coefficients involving viscosity occur in terms containing derivatives of the highest order in the system of differential equations, and thus developments in the neighborhood of zero viscosity involve boundary layer effects because of the loss of order of the differential equations in the limit. The terms involving compressibility occur, on the other hand, in a quite innocuous fashion.

The second chapter of the book is concerned with problems involving liquids with free boundaries and a study of jets, wakes, cavities behind solids moving through liquids, the theoretical explanation of the action of shaped charges (in which subject the author himself has made a basic contribution), and other like topics. The mathematical theory fundamental for these problems is in the main the classical Kirchhoff-Helmholtz theory, and it is given a brief but readable presentation. The recent literature on this subject is referred to very extensively in this chapter, including much material that is not well known. This chapter of the book should be very useful.

The third chapter is concerned with modeling and dimensional analysis. The reviewer found it interesting reading. The discussion of modeling, which is carried out on the basis of concrete examples, was particularly well done.

The fourth and fifth chapters, entitled *Group theory and fluid mechanics*, and *Virtual mass and groups* respectively, are, as the titles indicate, concerned with various roles which the concept of a group can play in fluid mechanics. For example, in chapter four the connection of transformation groups expressing various symmetries with the integration theory of partial differential equations is discussed.

J. J. STOKER

Introduction à la géométrie non euclidienne par la méthode élémentaire. By G. Verriest. Paris, Gauthier-Villars, 1951. 8+193 pp. 1000 fr.

This exposition begins with the foundations of Euclidean geometry: Hilbert's axioms of incidence, order, congruence, and parallelism. Setting aside the axiom of parallelism, the author develops a number of theorems belonging to both Euclidean and hyperbolic geometry. He takes care to avoid any appeal to intuition; for example, he proves in detail that the sum of two supplementary angles is equal to the sum of two right angles. This part of the work, making no assumption of continuity, culminates in the "second theorem of Legendre": If, in at least one triangle, the angle-sum is equal to two right angles, the same holds for every triangle. In Chapter IV, Dedekind's axiom of continuity is used to prove the axiom of Archimedes and the "first theorem of Legendre": The angle-sum of a triangle cannot exceed two right angles. The author mentions that this was anticipated by Saccheri, and that it would not hold without continuity [Dehn, Math. Ann. vol. 53 (1900) pp. 436–439]. Chapter V establishes the equivalence, in the presence of the other axioms, of three possible formulations of the Euclidean axiom: Euclid's own Postulate V, the unique parallel through a given point to a given line, and the angle-sum of a triangle being equal to two right angles. It follows that all three are contradicted in hyperbolic geometry.

The next three chapters are on Hilbert's treatment of area and its connection with angle-sum. Chapter IX deals with simple properties of parallels and ultraparallels in the hyperbolic plane. On p. 142 the author omits, as being "assez compliquée," the proof of the existence of a common parallel line to two given rays [Hilbert, *Grundlagen der Geometrie*, Leipzig, 1913, p. 151; Coxeter, *Non-Euclidean geometry*, Toronto, 1947, p. 205]. He excuses the omission by declaring that this result will not be required later; but he actually uses it on p. 150 and again on p. 156, each time assuring the reader that it will not be needed any more! There is a very readable account of circles, horocycles, and hypercycles, with a short paragraph on the extension to three dimensions.

Elliptic geometry is relegated to a single chapter at the end. The statement that every two coplanar lines intersect is shown to imply that each line of the plane has one or two absolute poles. The two alternative hypotheses are carefully examined, with appropriate