THE APRIL MEETING IN CHICAGO

The four hundred sixty-ninth meeting of the American Mathematical Society was held at the University of Chicago on Friday and Saturday, April 27–28, 1951. There were about 225 registrations including the following 205 members of the Society:

W. R. Allen, Nachman Aronszajn, W. L. Ayres, G. E. Backus, Reinhold Baer, W. R. Ballard, E. W. Banhagel, W. E. Barnes, R. G. Bartle, P. T. Bateman, A. F. Bausch, J. H. Bell, L. D. Berkovitz, Gerald Berman, S. F. Bibb, R. H. Bing, K. E. Bisshopp, H. D. Block, R. P. Boas, D. G. Bourgin, G. F. Bradfield, Joseph Bram, A. H Brown, R. C. Buck, P. B. Burcham, R. S. Burington, Eugenio Calabi, A. P. Calderón, R. E. Carr, A. B. Carson, Lamberto Cesari, S. S. Chern, Joshua Chover, H. M. Clark, R. A. Clark, M. D. Clement, B. H. Colvin, E. G. H. Comfort, Max Coral, J. J. Corliss, D. S. Curry, M. L. Curtis, Allen Devinatz, A. H. Diamond, Flora Dinkines, J. L. Doob, W. F. Eberlein, M. H. M. Esser, H. P. Evans, R. L. Evans, Trevor Evans, H. S. Everett, G. M. Ewing, Chester Feldman, W. H. Fleming, L. R. Ford, Abraham Franck, J. C. Freeman, C. G. Fry, R. E. Fullerton, J. W. Gaddum, M. P. Gaffney, B. A. Galler, Murray Gerstenhaber, Michael Golomb, S. H. Gould, L. M. Graves, R. E. Graves, V. G. Grove, M. M. Gutterman, Franklin Haimo, P. R. Halmos, Frank Harary, W. L. Hart, Charles Hatfield, L. L. Helms, R. G. Helsel, Melvin Henriksen, I. N. Herstein, D. G. Higman, J. G. Hocking, Carl Holtom, T. C. Holyoke, W. A. Howard, H. K. Hughes, W. E. Jenner, A. K. Jennings, Meyer Jerison, R. N. Johanson, L. W. Johnson, G. K. Kalisch, Wilfred Kaplan, William Karush, L. M. Kelly, D. E. Kibbey, S. C. Kleene, Erwin Kleinfeld, J. C. Koken, Jacob Korevaar, M. Z. Krzywoblocki, R. G. Kuller, H. G. Landau, E. P. Lane, R. E. Langer, Leo Lapidus, J. R. Lee, O. I. Litoff, Lee Lorch, S. W. McCuskey, A. W. McGaughey, R. W. MacDowell, Saunders MacLane, H. B. Mann, H. G. Mazurkiewicz, D. M. Merriell, B. E. Meserve, H. L. Meyer, E. A. Michael, C. E. Miller, J. M. Miller, J. M. Mitchell, C. W. Moran, E. J. Moulton, E. D. Nering, E. A. Nordhaus, R. Z. Norman, C. O. Oakley, Rufus Oldenburger, H. W. Oliver, E. H. Ostrow, R. R. Otter, G. K. Overholtzer, Gordon Pall, S. T. Parker, M. H. Payne, M. M. Peixoto, George Piranan, D. H. Potts, A. L. Putnam, Gustave Rabson, L. I. Rebhun, O. W. Rechard, R. F. Reeves, W. T. Reid, Haim Reingold, Daniel Resch, R. A. Roberts, Alex Rosenberg, Murray Rosenblatt, Arthur Rosenthal, E. H. Rothe, M. F. Ruchte, R. G. Sanger, L. J. Savage, O. F. G. Schilling, Lowell Schoenfeld, A. L. Schurrer, W. R. Scott, W. T. Scott, I. E. Segal, R. J. Semple, D. H. Shaftman, Daniel Shanks, M. E. Shanks, Edward Silverman, R. J. Silverman, Annette Sinclair, M. L. Slater, D. M. Smiley, M. F. Smiley, A. H. Smith, Albert Soglin, E. S. Sokolnikoff, E. H. Spanier, E. J. Specht, G. L. Spencer, W. L. Stamey, R. G. Stanton, H. E. Stelson, B. M. Stewart, M. H. Stone, L. W. Swanson, Otto Szász, C. T. Taam, T. T. Tanimoto, H. P. Thielman, G. H. M. Thomas, G. L. Thompson, C. J. Titus, E. A. Trabant, E. F. Trombley, H. L. Turrittin, E. H. Umberger, G. L. Walker, D. R. Waterman, George Whaples, J. J. Wheeler, L. R. Wilcox, V. M. Wolontis, Oswald Wyler, L. C. Young, P. M. Young, J. W. T. Youngs, R. K. Zeigler, Daniel Zelinsky.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, Professor I. E. Segal spoke on *Noncommuta-*

tive integration and measurable operators at 2:00 P.M. Friday. Professor M. H. Stone presided at the session.

Sessions for the presentation of contributed papers were held at 10:30 A.M. and 3:15 P.M., Friday, and 10:30 A.M. on Saturday. Presiding officers at these sessions were Professors Reinhold Baer, G. M. Ewing, Saunders MacLane, E. H. Spanier, and D. G. Bourgin. Relative to contributed papers, a happy note is that the time limit of ten minutes is coming more to be accepted as an upper bound, rather than a lower bound as has so frequently been the case in the past.

The list of papers presented follows. Those abstracts indicated by the letter "*t*" were presented by title. Mr. George Klein was introduced by Professor Antoni Zygmund.

ALGEBRA AND THEORY OF NUMBERS

325. Melvin Henriksen: *Maximal ideals of the ring of entire functions and their residue class fields.* Preliminary report.

Let R denote the ring of entire functions, M any maximal ideal. The latter are classified and it is shown that R/M is always isomorphic to the complex field K (as a ring but not as an algebra). No topological restrictions are placed on the maximal ideals M. The method of proof is to show that (i) R/M is algebraically closed, (ii) R/M has the cardinal number of the continuum. Together with a theorem of Steinitz, (i) and (ii) imply that R/M is isomorphic to K. This result can be extended to other function rings. (Received April 10, 1951.)

326. I. N. Herstein: *A generalization of a theorem of Jacobson.*

The result obtained in this paper is as follows: Let R be a ring with a unit element, and center Z. If there exists an integer $n>1$ so that $x^n-x\in Z$ for all $x\in R$, then R is commutative. The method of proof is to first prove the result for division rings and primitive rings, and so obtain it for all semi-simple rings. In the case that the ring has a radical, this enables us to locate precisely the commutators and the radical. From this information the result can be obtained for subdirectly irreducible rings, and so for all the rings under consideration. It is conjectured that the result remains true if n is not fixed but $n=n(x)$, and even if the ring does not have a unit element. (Received February 23, 1951.)

327. D. G. Higman: *Lattice homomorphisms induced by group homomorphisms.*

A single-valued mapping ϕ of the lattice $L(G)$ of all subgroups of a group G onto the lattice $L(H)$ of all subgroups of a group H will be called a *lattice homomorphism of G onto H* if (1) $(\bigcup_\nu S_\nu)\phi=\bigcup_\nu(S_\nu\phi)$ and (2) $(\bigcap_\nu S_\nu)\phi=\bigcap_\nu(S_\nu\phi)$ for every system of subgroups S_ν of G. ϕ will be called *proper* if it is neither 1-1 nor trivial. Every homomorphism of a group G onto a group H induces a single-valued mapping of $L(G)$ onto $L(H)$ which satisfies (1). The main result of this note is a characterization of those groups which admit *homomorphisms* which induce *proper lattice homomorphisms*. Our results contain those obtained by G. Zappa for finite groups (Giornale di Matematiche di Battaglini (4) vol. 78 (1949) pp. 182–192). (Received March 5, 1951.)

328t. Erwin Kleinfeld: *On the structure of simple alternative rings without chain conditions.* Preliminary report.

Let the alternative ring R be simple, not associative, without nilpotent elements and of characteristic not 2. Then it follows that R is a Cayley-Dickson division algebra. Refinements of methods from a paper by R. H. Bruck and the author (See Bull. Amer. Math. Soc. Abstracts 56-5-446 and 56-6-453) are employed. (Received February 5, 1951.)

329. E. D. Nering: *On the extensions of the field of constants of an algebraic function field.*

Let k be any field whatsoever, and let K be an algebraic function field of one variable of genus p over k. If Λ is transcendental over K, then $K(\Lambda)$ is an algebraic function field of genus p over $k(\Lambda)$. If λ is algebraic over k, let $K(\lambda)$ be the Kronecker product of K and $k(\lambda)$, and express $K(\lambda)$ as a direct sum of completely primary rings, $K(\lambda)=K(\lambda)_1+ \cdots +K(\lambda)_s$. Let Γ_i be the radical in $K(\lambda)_i$, and let the radical degree of $K(\lambda)_i$ over $K_i=K(\lambda)_i/\Gamma_i$ be e_i. If p_i is the genus of the algebraic function field K_i over (the isomorphic image of) $k(\lambda)$, then $p-1=\sum_{i=1}^{s} e_i(p_i-1)+\rho$ where $\rho \geqq 0$ is one-half the order of the norm of a conductor in $K(\lambda)$. (Received February 19, 1951.)

330. Alex Rosenberg: *Subrings of simple rings with minimal ideals.*

The following theorems are proved: (1) If a π-regular ring is atomic modulo its radical, so is every π-regular subring. (2) Let $\mathfrak{A}$ be a simple ring with a minimal left ideal (S.M.I. ring). If $\mathfrak{B}$ is a π-regular simple subring not satisfying the minimum condition, $\mathfrak{B}$ is again S.M.I. and $\mathfrak{A}(\mathfrak{B})$, the commutator of $\mathfrak{B}$ in $\mathfrak{A}$, is the two-sided annihilator of $\mathfrak{B}$ in $\mathfrak{A}$. $\mathfrak{A}(\mathfrak{B})$ may have a radical which is a zero ring, and $\mathfrak{A}(\mathfrak{B})-\mathfrak{R}$ is again S.M.I. (3) Let $\mathfrak{A}$ be a dense algebra of finite-valued linear transformations in a vector space $\mathfrak{M}$ over a field. Let $\mathfrak{B}$ and $\mathfrak{C}$ be simple, isomorphic, π-regular subalgebras. Let $\mathfrak{M}\mathfrak{B}$ and $\mathfrak{T}_{\mathfrak{B}}$ denote the range and null space of $\mathfrak{B}$ in $\mathfrak{M}$ respectively; $\mathfrak{M}\mathfrak{B}\cap\mathfrak{T}_{\mathfrak{B}}=0$. Then the isomorphism between $\mathfrak{B}$ and $\mathfrak{C}$ can be extended to an inner automorphism of the entire algebra of linear transformations on $\mathfrak{M}$ if (i) $\mathfrak{A}(\mathfrak{B})$ is isomorphic to $\mathfrak{A}(\mathfrak{C})$, (ii) $\mathfrak{M}\mathfrak{B}$ and $\mathfrak{M}\mathfrak{C}$ break up into equipotent families of irreducible $\mathfrak{B}$ and $\mathfrak{C}$ modules, (iii) $\mathfrak{M}=\mathfrak{M}\mathfrak{B}\oplus\mathfrak{T}_{\mathfrak{B}}=\mathfrak{M}\mathfrak{C}\oplus\mathfrak{T}_{\mathfrak{C}}$. (Received March 14, 1951.)

331. W. R. Scott: *Groups and cardinal numbers.*

Let G be an infinite group with identity e. For $x\in G$, let $E(x)$ be the set of $g\in G$ such that the equation $g^n=x$ has no solutions for n. Let K be the set of $k\in G$ such that $o(E(k))<o(G)$ where $o(S)$ is the cardinal number of elements in S. Let D be the intersection of all subgroups G_α of G with $o(G_\alpha)=o(G)$. It is shown that $K\subseteq D$ and that K is a central, fully characteristic subgroup of G, which is either a cyclic group of order p^n or a p^∞ group. For Abelian groups G, $K=D=H$ if $G=H\times F$ where H is a p^∞ group and F is finite, and $K=D=e$ otherwise. A layer $L(n)$ $(L(\infty))$ of G is the set of elements of order n (∞). Several results are obtained concerning the size of layers of infinite groups. In particular, if G is not periodic, then $o(L(\infty))=o(G)$. If G is Abelian and nondenumerable, then there are $2^{o(G)}$ subgroups G_α of G with $o(G_\alpha)=o(G)$. (Received January 18, 1951.)

332. Daniel Zelinsky: *Complete fields without valuations.*

There exists complete topological fields other than those complete in a valuation. A class of such fields is obtained as follows. Let R be any integral domain not a field,

F its quotient field. The R-topology on F is defined by designating the nonzero ideals in R as the neighborhoods of zero [Bull. Amer. Math. Soc. vol. 54 (1948) pp. 1145–1150]. Usually F is neither complete nor completable. However, if R is a local ring complete in the local topology, then F is complete in the R-topology. If R is any complete local ring with dimension greater than one and no divisors of zero (for example, the ring of power series in n variables, $n>1$), its quotient field, in the R-topology, is a complete field whose topology is not given by a valuation. (Received February 1, 1951.)

Analysis

333. Nachman Aronszajn: *A characterization of commutative types of order.*

The following theorem is proved: For two types of order α and β, a necessary and sufficient condition in order that $\alpha+\beta=\beta+\alpha$ is that at least one of the following representations be valid. (i) $\alpha=\beta\omega+\gamma+\beta\omega^*$, with some type of order γ, (i′) $\beta=\alpha\omega+\gamma+\alpha\omega^*$, with some type of order γ, (ii) $\alpha=\sum_{0\leqq\xi<\xi_0}\gamma(\xi)$, $\beta=\sum_{\xi_0\leqq\xi<1}\gamma(\xi)$, where ξ_0 is some number greater than 0 and less than 1; $\gamma(\xi)$ is a function defined for $0\leqq\xi<1$, having for values types of order and satisfying the condition $\gamma(\xi_1)=\gamma(\xi_2)$ for any ξ_1, ξ_2 for which there exist integers m, m_0, such that $\xi_1-\xi_2=m+m_0\xi_0$. (Received March 12, 1951.)

334. Nachman Aronszajn and A. K. Jennings: *A new approximation method for partial differential eigenvalue problems.* Preliminary report.

The method approximates the eigenvalues by upper bounds. The approximations are obtained as eigenvalues of systems of ordinary differential equations. (Received March 12, 1951.)

335. R. C. Buck: *Operator algebras and dual spaces.*

Let X be a translation invariant linear space of functions on a group G, and $B(X)$ be its associated algebra of linear operators. Let G_0 be the group of left translation operators, induced by G on X. Then, under suitable conditions, the centralizer B^0 of G_0 in $B(X)$ is isomorphic as a linear space to the dual space X' of X. The operator multiplication then induces a convolution product in X'. This has applications to the study of group algebras, and to the theory of distributions (Schwartz, Actualités Scientifiques et Industrielles, Paris, 1950). (Received March 12, 1951.)

336. R. H. Cameron and Charles Hatfield: *On the summability of certain series for unbounded nonlinear functionals.*

In a previous paper (*On the summability of certain orthogonal developments of nonlinear functionals*, Bull. Amer. Math. Soc. vol. 55 (1949) pp. 130–145) the authors showed that the Fourier-Hermite expansion (Ann. of Math. vol. 48 (1947) pp. 385–392) of a bounded Wiener measurable (nonlinear or linear) functional is Abel summable (in an appropriate infinite-dimensional sense) to the value of the functional at each point where the functional is continuous in the Hilbert topology. In the present paper the authors remove the condition of boundedness on the functional and assume a functional $F(x)$ such that $|F(x)|\leqq B\exp\{A\int_0^1[x(t)]^2dt\}$. (Received March 13, 1951.)

337t. Harvey Cohn: *A conformal mapping property of the Eisenstein series.* Preliminary report.

The upper half z-plane is mapped on the upper half w-plane with slits running from $w=p/q$ to $w=p/q+i\epsilon/q^n$. The mapping function is given for small ϵ approximately by $w=z+\epsilon^2\phi_n(z)$ where $d^{(2n-1)}\phi_n(z)/dz^{2n-1}$ is (essentially) the Eisenstein series $\sum(pz+q)^{-2n}$. For $n=1$ a slight modification of the limit $(\epsilon\to 0)$ produces the partition function. The method used is a perturbation method, which brings out a type of infinitesimal invariance in the mapping function a priori. (Received March 6, 1951.)

338. R. L. Evans: *Solution in the large of linear differential equations.*

W. B. Ford's method of solution in the large is extended to equations whose rank at an irregular singular point exceeds unity and can first be reduced by Poincaré's method. Also, the treatment of Ford's case 2 of type III is simplified and made to give more explicit results by a change of the dependent variable that simultaneously replaces the solution of a certain second order difference equation with that of a first order difference equation. As a different development, a simple but apparently new method of solution in the large is presented which utilized convergent solutions about an irregular singular point, usually at infinity. These involve series of inverse factorials and were supplied for many differential equations by J. Horn and W. J. Trjitzinsky. Another method of getting such solutions and a new criterion for their convergence are described. These suggest a change of the independent variable that makes possible the solution in the large of some previously unsolvable linear differential equations. Several examples are given, including one previously unsolvable and two previously unsolved equations. (Received March 14, 1951.)

339. R. P. Gosselin and L. D. Berkovitz: *On the theory of localization for double trigonometric series.*

In this paper the authors consider the problem of localization for double trigonometric series $\sum a_{mn} \exp i(mx+ny)$ summed by the restricted method. Let the coefficients a_{mn} be $o(|m|^{\gamma}|n|^{\delta})$, $\gamma>-1$, $\delta>-1$, and set $\beta=\gamma+\delta+1$. If the series is formally integrated with respect to x and y sufficiently many times to produce uniform convergence to a function $F(x, y)$, and if $\lambda(x, y)$ is a sufficiently good localizing function associated with the rectangles R' and R, $R'\subset R$, then $\Delta_{M,N}(x, y)=\sum a_{m,n}e^{i(mx+my)} -((-1)^{K+L}/\pi^2)\int_{-\pi}^{+\pi}\int_{-\pi}^{+\pi}F(u, v)\lambda(u, v)D_M^K(x-u)D_N^L(y-v)dudv$ is (C, β, β) restrictedly summable to zero, uniformly for (x, y) in R', where $D_M^K(u)$ denotes the kth derivative of the Dirichlet kernel of order M. From the above Riemann formula one obtains a localization principle involving only ordinary neighborhoods for double trigonometric series summed restrictedly. The method of proof involves the use of formal multiplication of double trigonometric series in a manner analogous to that used by Rajchman and Zygmund for the one variable case. An extension to two variables of Fatou's theorem on power series is obtained as a by-product of the above theory. (Received March 15, 1951.)

340. Wilfred Kaplan: *On Gross's star theorem, schlicht functions, logarithmic potentials, and Fourier series.*

Gross's star theorem asserts that each element of the inverse of a meromorphic function $g(z)$ can be continued to infinity along "almost all rays" from the center of the element. This theorem is generalized and connections with the theory of schlicht functions, logarithmic potentials, and Fourier series are shown: 1. The Gross theorem remains valid if the family of rays is replaced by any conformal image of a family of parallel lines. 2. Gross's theorem and 1. remain valid if $g(z)$ is replaced by any function $G(z)$ which satisfies an equation $f(G(z))=h(z)$, where f and h are meromorphic for finite z. 3. Let $\mathcal{E}$ be the class of all closed sets E on $|z|=1$ for which there exists a schlicht function $f(z)$ in $|z|<1$ such that f becomes infinite for arbitrary approach to each point of E. Then $\mathcal{E}$ coincides with the class of closed sets of logarithmic capacity 0 on $|z|=1$. 4. The sum of the conjugate trigonometrical series $\bar{S}[f]$ of a function $f(\theta)$ of bounded variation can be represented as a logarithmic potential of a mass distribution of variable sign on $|z|=1$. 5. The class of closed sets E on $|z|=1$ for which there exists an $f(\theta)$ of bounded variation such that $\bar{S}[f]$ diverges for exp $(i\theta)$ in E is the class $\mathcal{E}$. (Received March 7, 1951.)

341. George Klein: *On the divergence of the Lagrange polynomials interpolating analytic functions.*

Let B be the Banach space of functions analytic interior to and continuous on the closed unit disc in the complex plane, with the norm of an element f of B defined as $\|f\|=\sup_{|z|=1}|f(z)|$. Let B_n be the subspace of B consisting of polynomials in z of degree at most n. Let I_n be the linear operator acting on B which maps an element f of B into the polynomial of degree n which agrees with f at $n+1$ points equally distributed on the circumference of the unit circle. The following two theorems are established: I. *Suppose that, for each n, L_n is a linear operator mapping B_n into B. Then for an arbitrary such sequence, the sequence L_nI_n of linear operators mapping B into B does not converge strongly to the identity mapping on B.* II. *Suppose that, for each n, L_n is the linear operator defined on B which maps each function $f(z)$ in B into the function $f_n(z)=((n+1)/2\pi)\int_{-\pi/(n+1)}^{\pi/(n+1)}f(ze^{it})dt$. Then the sequence of composite linear operators I_nL_n defined on B converges strongly to the identity operator acting on B.* (Received April 27, 1951.)

342. Lee Lorch: *On derivatives of infinite order.*

Some of the results derived by Boas and Chandrasekharan concerning $\lim f^{(n)}(x)$ as n becomes infinite (Bull. Amer. Math. Soc. vol. 54 (1948) pp. 523–526, 1191) are restudied with " lim" taken to mean summation by Borel's exponential means instead of convergence. Additional remark: Following their notation, it is pointed out that if the lim inf occurring in their theorem 3(ii) (loc. cit. p. 525) is not the same as lim sup (that is, if the limit does not exist), then $g(x)\equiv 0$ in $a\leqq x\leqq b$. (Received March 14, 1951.)

343. Josephine M. Mitchell: *The Bergman kernel function in the geometry of matrices.*

We consider the domain D defined by $Z\bar{Z}'<I$, where Z is a rectangular matrix of complex numbers z_{jk} $(j=1, \cdots, m;\ k=1, \cdots, n)$, $\bar{Z}'$ its conjugate transpose, and I the identity matrix. It is proved by means of a minimal problem that the function $K(Z, \bar{W}')=V^{-1}[\det(I-Z\bar{W}')]^{-m-n}$, where Z, $\bar{W}'\in D$ and V is the Euclidean volume of D, is the Bergman kernel function of the domain D. From this follows the symmetry property $K(Z, \bar{W}')=\overline{K(W, \bar{Z}')}$ and the reproducing formula $f(Z)=\int\cdots\int_D K(Z, \bar{W}')f(W)dV$ $(Z\in D)$ for any function $f(Z)$, analytic in $z_{11}, \cdots, z_{mn}$, such that $\int\cdots\int_D|f(Z)|^2dV$ is finite. Similar results hold for the cases where Z is a symmetric or skew-symmetric matrix. (Received March 12, 1951.)

344. George Piranian: *Comparison of the collective Hausdorff method with the method of essential Hausdorff cores.*

A sequence s is evaluated to the finite value p by the collective Hausdorff method $\mathcal{H}$ [Agnew, Trans. Amer. Math. Soc. vol. 52 (1942) pp. 217–237] provided there exists a regular Hausdorff matrix A such that the sequence As converges to p. If s is bounded, the core $C(s)$ is defined as the least convex set containing all limit points of s; the definition has been extended in an appropriate manner to deal with unbounded sequences. The essential Hausdorff core of s is the set H^*s consisting of the intersection of all cores $C(As)$, where the symbol A ranges over all completely regular Hausdorff matrices. The author shows that if s is evaluated to p by $\mathcal{H}$, then the set H^*s is either unbounded or consists of the single point p. On the other hand, he exhibits a bounded sequence s such that the set H^*s consists of the origin while the method $\mathcal{H}$ does not evaluate s. (Received March 13, 1951.)

345. O. W. Rechard: *A note on the summability of infinite series by sequence to sequence and series to sequence transformations.*

If the infinite matrix $A=(a_{ij})$ defines a regular sequence to sequence method of summability, it is well known that the matrix $B=(b_{ij})$ obtained by defining $b_{ij} = \sum_{n=j}^{\infty} a_{in}$ defines a regular series to sequence method of summability. Moreover, a series with bounded partial sums is summed by A to s if and only if it is summed by B to s. However, if $B=(b_{ij})$ determines a regular series to sequence method of summability and $A=(a_{ij})$ is defined by $a_{ij}=b_{ij}-b_{i,j+1}$, then the sequence to sequence method of summability determined by A need not even be regular. It is the purpose of this note to point out that this apparent superiority of series to sequence methods of summability over sequence to sequence methods is really illusory. In fact, if $B=(b_{ij})$ determines a regular series to sequence method of summability which sums some divergent series with bounded partial sums, then the corresponding sequence to sequence method A, as defined above, is regular, and a series with bounded partial sums is summed by B to s if and only if it is summed by A to s. (Received March 9, 1951.)

346. R. F. Reeves: *A center of gravity problem.* Preliminary report.

Let r be the position vector in a three-dimensional Euclidean space, and let $mF(r)$ be the force exerted on a particle of mass m placed at r. Assume $F(r)$ to be continuous and to admit of centers of gravity in the following sense. For every finite system of particles of mass $m_1, m_2, \cdots, m_n$ located at $r_1, r_2, \cdots, r_n$ respectively, there exists a point $r_c(r_1, r_2, \cdots, r_n)$, called the center of gravity of the system, which satisfies the following conditions: (a) $\sum_{i=1}^{n} m_i F(ri) = (\sum_{i=1}^{n} mi) F(r_c)$, (b) $r_c = \sum_{i=1}^{n} \alpha_i r_i$, where $\alpha_1, \alpha_2, \cdots, \alpha_n$ are real numbers, (c) $r_c(Or_1+b, Or_2+b, \cdots, Or_n+b) = Or_c(r_1, r_2, \cdots, r_n)+b$, where O is an arbitrary orthogonal matrix and b any constant vector. Then $\sum_{i=1}^{n} \alpha_i = 1$, and $F(r)=Ar+c$, A being an arbitrary constant matrix and c being any fixed vector. In the paper *On fields of force in which centers of gravity can be defined* by John Aczél, and Stephen Fenyö, Hungarica Acta Mathematica vol. 1 (1948), a similar problem was considered, conditions (b) and (c) being different. (Received March 9, 1951.)

347. Edward Silverman: *Set functions associated with Lebesgue area.*

If x is a continuous function defined on a square Q into E_n, let $P(x)$, $L(x)$, and $x(Q)$ denote the Peano area, the Lebesgue area, and the range of x, respectively. There is an analytic procedure, independent of the notion of Lebesgue area, for constructing a monotone function y on Q into m, the space of bounded sequences, such that $L(x)=L(y)$. Then there are defined two, possibly different, non-negative set functions λ and μ, with domain the subsets of m, which give the elementary area for elementary configurations and satisfy the following statements: (i) If $P(x)=L(x)$, then $\lambda(y(Q))=L(x)$, and (ii) $\mu(y(Q))=L(x)$. (Received March 12, 1951.)

348. Annette Sinclair: *Approximations and the Weierstrass-factor and Mittag-Leffler theorems.*

In this paper the existence of a function is shown which—in addition to satisfying the conditions required of the function constructed in the Wierstrass-factor (or Mittag-Leffler) Theorem—satisfies a preassigned approximation condition in a certain neighborhood of each zero (or pole). Let $M(z)$ be any function which is analytic and not identically zero in a neighborhood of each point of an isolated set J and which vanishes at each point of J. It is proved that there exists a function $h(z)$ such that: (1) $h(z)$ is analytic in the entire plane except at limit points of J; (2) $h(z)$ has zeros at precisely the points of J of the same orders as those of $M(z)$; (3) $h(z)$ approximates $M(z)$ arbitrarily closely in a certain preassigned neighborhood of each point of J. An analogous generalization of the Mittag-Leffler Theorem is given in which an approximation condition is preassigned in a certain deleted neighborhood of each pole. In either generalization the closeness of approximation may be preassigned independently for neighborhoods of different points of the isolated set on which zeros (or poles) are required. (Received March 9, 1951.)

349. Otto Szász: *On products of summability methods.*

Suppose that A and B are two regular summability methods for sequences $\{S_n\}$ or series $\sum a_n$. Denote by AB the iteration product which associates with a given sequence the A transform of its B transform. Particular cases of the problem are discussed: when does summability A imply summability (AB)? The answer is shown to be in the affirmative in the following cases: (a) B is (C, α) and A is either Abel or Borel summability; (b) B is Riesz summability and A is the Laplace transform; (c) B is Euler- and A is Borel-summability. The special case $\alpha=1$ has been discussed before. (Received March 7, 1951.)

350. H. L. Turrittin: *Asymptotic expansions of solutions of systems of ordinary linear differential equations containing a parameter.*

W. J. Trjitzinsky has foreseen that the system of differential equations (1), $\epsilon^h dX/dt=A(t, \epsilon)X(t)$, where h is a positive integer; ϵ is a small parameter; X is a column vector with components $x_1(t), \cdots, x_N(t)$; and where the square matrix $A(t, \epsilon)$ has a convergent expansion $A=\sum_{i=0}^{\infty}\epsilon^i A_i(t)$ in the domain $|\epsilon|\leqq\epsilon_1$, $t_1\leqq t\leqq t_2$, possesses N formal vector solutions each of the form $X=\sum_{j=0}^{\infty}\epsilon^{j/r}X_j(t)\cdot\exp\{\int_{t_1}^{t}\epsilon^{-h}\rho(\tau, \epsilon)d\tau\}$ where the X_j are vectors; r is a suitable positive integer; and $\rho(t, \epsilon)=\sum_{j=0}^{hr-1}\epsilon^{j/r}\rho_j(t)$ where the $\rho_j(t)$ are suitably chosen functions of t (see Acta Math. vol. 67 (1936) pp. 1–50). In this reference Trjitzinsky also proves under suitable hypotheses that, *if such formal solutions exist*, they represent asymptotic solutions of (1). In the present paper a straightforward method for evaluating the $\rho_j(t)$'s, r, and $X_j(t)$'s in the formal solutions is presented, based on a sequence of nonsingular matrix transformations of four types: $X=P(t)Y$; $X=Y\exp\{\int_{t_0}^{t}\epsilon^{-h}\rho(\tau)d\tau$;

$X=[I+\epsilon^k Q_k(t)]Y$ where I is the unit matrix; and $x_i(t)=\epsilon^{(N-i)k}y_i(t)$ $(i=1, \cdots, N)$, where k is a positive integer or fraction. (Received February 6, 1951.)

Applied Mathematics

351. M. R. Hestenes and William Karush: *The solutions of $Ax=\lambda Bx$.*

Let A, B be Hermitian matrices of order n with B positive definite. An iterative gradient method is studied for determining the characteristic values and vectors of the equation $Ax=\lambda Bx$. The method stems from the fact that the characteristic vectors are the critical points of the "Rayleigh quotient" $\mu(x)=(x, Ax)/(x, Bx)$. One passes from x to the next approximation x' by means of the formula $x'=x-\alpha\eta$ where $\alpha>0$ and $G\eta=Ax-\mu x$ (G is an arbitrary fixed positive definite matrix with known inverse). It is shown that with appropriate choice of α the iteration converges to a characteristic vector, and, in some cases, to the vector with least characteristic value. The procedure can be adapted to yield all the characteristic vectors; it has the advantage of avoiding transformation of the problem and of being mechanizable. Finally it is shown that a general problem $Cx=\lambda Dx$ (C, D arbitrary matrices with $|C-\lambda D|\not\equiv 0$) is equivalent to one of the above type if and only if there exists a positive definite Hermitian matrix P such that CPD^* (or C^*PD) is Hermitian (D^* = conjugate transpose of D). (Received March 5, 1951.)

352. M. Z. Krzywoblocki: *Extension of Kampé de Fériet's formula on kinetic energy in simply connected domain to compressible fluid.*

Leray and subsequently Kampé de Fériet derived and proved the exponential form of the function expressing the decrease of kinetic energy of an incompressible viscous fluid flow in a simply connected domain without any external forces to resist or to excite the motion. In the present note the author derives such a form for a compressible viscous fluid flow and shows that the formulae obtained by previous authors are only particular cases of the more general forms derived by him. (Received February 23, 1951.)

353t. Daniel Resch: *Temperature bounds on the infinite rod.*

It is desired to know the bounds on the positive temperature, $u(x, t)$, of an infinite rod at any time ($t>0$) and at any point of the rod when the temperature is known at certain points and at certain times. While the problem is treated by P. C. Rosenbloom, *Quelques classes de problèmes extrèmaux* (to appear in Bull. Soc. Math. France, 1951), the author employs simple methods to show that if the temperature is known at one point and time, $u(x_0, t_0)=c$, then $u(x, t)\leqq cH(x, t)$ for $0<t\leqq t_0$ and the inequality reversed for $0<t_0\leqq t$ where $H(x, t)=(t_0/t)^{1/2}\exp[(x-x_0)^2/4(t_0-t)]$. With the aid of the first inequality, the author proves an analog of Harnack's second theorem; namely, that a monotone increasing sequence of solutions of the heat equation bounded at one point (x_0, t_0) converges uniformly in the domain $|x-x_0|\leqq A(t_0-t)^{1/2}$, $t\geqq\epsilon>0$ (A, ϵ constants). (Received March 13, 1951.)

Geometry

354. Eugenio Calabi: *Isometric, complex analytic imbedding of complex manifolds.*

The problem considered is whether a given complex manifold M^k with any metric admits an isometric, complex analytic imbedding in a Fubini-Study space of dimension $N(\leqq \infty)$ and constant curvature b on holomorphic sections (see Bochner, Bull. Amer. Math. Soc. vol. 53 (1947) pp. 179–195). A first necessary condition is that the metric on M^k be an analytic, Kähler metric. The latter gives rise by integration to a real analytic function of pairs of points, called their *diastasis*, which is asymptotically (as the points become close) the square of their geodesic distance. The diastasis, in contrast with geodesic distance, has the property that, if the two points lie on a complex submanifold, their diastasis with respect to the metric induced on the submanifold is the same as that obtained from the ambient space. This implies that any isometric, complex analytic imbedding is determined up to within a motion in the ambient space. The necessary and sufficient conditions for the existence of an isometric, complex analytic imbedding locally are then deduced in terms of the coefficients of the power series expansion of the diastasis; the imbedding is global if the diastasis is single-valued. (Received March 14, 1951.)

355. Mary Dean Clement: *A criterion for determining the space of immersion of a variety of arbitrary dimensionality.*

This paper establishes a necessary and sufficient condition that a proper analytic variety V_m: $x=x(u_1, \cdots, u_m)$, defined in a projective linear space S_n, be immersed in a linear subspace S_k. Relative to a given variety V_m let the set of matrices $M_m^{(h)}$ $(h=0, 1, 2, \cdots)$ be defined recursively: $M_m^{(0)}\equiv(x)$; $M_m^{(h)}(h\geqq 1)$ is formed from $M_m^{(h-1)}$ by adjoining as columns all partial derivatives of x of order h. Consideration of properties of this set of matrices and consideration of geometric properties of V_m lead to the conclusion: a necessary and sufficient condition that a proper analytic variety V_m be immersed in a projective linear space S_k is that the matrix $M_m^{(k-m+2)}$ be of rank $k+1$. Interpretation of this result in terms of partial differential equations yields the equivalent result: a necessary and sufficient condition that V_m be immersed in S_k is that the coordinates of an ordinary point on V_m satisfy a completely integrable system of $C_{k+2,m}-(k+1)$ linearly independent linear homogeneous partial differential equations of order $k-m+2$ or less, of which precisely $C_{k+1,m-1}$ equations are actually of order $k-m+2$. Specialization of the latter condition when $m=1$ gives the classical criterion for the space of immersion of a proper analytic curve. (Received March 14, 1951.)

356. J. W. Gaddum: *Spread of an arc in a metric space.*

Let A be an arc in a metric space, with $p=f(x)$, $p\in A$, $x\in[0, 1]$, f a homeomorphism between A and the unit interval. If $\lim_{x,y\to t} f(x)f(y)/xy$ exists finite it is denoted $f'(t)$ and called the spread of f at t. $A(a, b)$ will denote the subarc of A from $f(a)$ to $f(b)$. If it is rectifiable its length will be denoted $s(a, b)$. The following theorems can be proved: (1) if $f'(t)$ exists on a set E, it is continuous on E; (2) if $f'(t)$ exists at each point of a, b, $A(a, b)$ is rectifiable and $s(a, b)=\int_a^b f'(t)dt$; (3) if $f'(t)$ exists at each point of $[a, b]$ and $f'(t_0)\neq 0$, $t_0\in[a, b]$, then $\lim_{x,y\to t_0} f(x)f(y)/s(x, y)=1$; (4) if A is rectifiable and f is the arc length parameterization and the chord arc ratio approaches 1 at t, then $f'(t)=s$. It may happen that $f'(t)$ fails to exist even though $\lim_{x\to t} f(x)f(t)/xt$ exists and is continuous at each point of $[0, 1]$. (Received March 12, 1951.)

Statistics and Probability

357. Murray Rosenblatt: *On the oscillation of sums of random variables.*

$X_1, X_2, \cdots$ are independent, identically distributed random variables with distribution function $F(x)$. $S_n=\sum_1^n X_j$. The sequence S_n is said to oscillate if $\Pr(S_n>0$ infinitely often$)=\Pr(S_n\leqq 0$ infinitely often$)=1$. If $\Pr(X_j\neq 0)>0$ and $E(|X_j|)<\infty$, oscillation takes place if and only if $E(X_j)=0$. $\limsup_{n\to\infty}\Pr(S_n>0)$, $\limsup_{n\to\infty}\Pr(S_n\leqq 0)>0$ is a sufficient condition for oscillation. When $E(|X_j|)=\infty$, it yields the following criterion for oscillation: If there are positive monotone sequences of integers $n_k\uparrow\infty$ and numbers $a_k\uparrow\infty$ such that (1) $\limsup_{k\to\infty} n_k/a_k<\infty$, (2) for some $\epsilon>0$ $\limsup_{k\to\infty}(n_k^2/a_k^2)\cdot\mathrm{Var}\,(X^2\,|\,|X|<a_k\epsilon)<\infty$, (3) $\lim_{k\to\infty} n_k(1-F(a_kx))=-\phi(x)$, $\lim_{k\to\infty} n_kF(-a_kx)=\phi(-x)$, $x>0$, where $\phi(x)$ is finite for $|x|\neq 0$ and increases somewhere in both ranges $x>0$ and $x<0$, then the S_n generated by X_j with distribution function $F(x)$ oscillate. $\sum_1^\infty\Pr(S_j>0,\ S_{j+1}\leqq 0)$, $\sum_1^\infty\Pr(S_j\leqq 0,\ S_{j+1}>0)=\infty$ is shown to be a necessary and sufficient condition for oscillation. It is used to obtain a class of random variables each of which generate sums S_n such that $\Pr(\lim_{n\to\infty}|S_n|=\infty)=\Pr(\limsup_{n\to\infty}S_n=\infty)=\Pr(\liminf_{n\to\infty}S_n=-\infty)=1$, $\lim_{n\to\infty}\Pr(S_n>0)=0$. (Received March 14, 1951.)

Topology

358. R. H. Bing: *A homeomorphism between the 3-sphere and the sum of two solid horned spheres.*

A horned sphere plus its bad complementary domain in S^3 is called a solid horned sphere. If two solid horned spheres are "sewn" together in a certain fashion, the question has been raised by R. L. Wilder as to whether or not the resulting set is homeomorphic with S^3. Suppose that the compact continuum M is the sum of three mutually exclusive sets U_1, H, and U_2 such that there is a homeomorphism of $H+U_1$ into the solid horned sphere that carries H into the horned sphere and there is a homeomorphism of $H+U_1$ into $H+U_2$ that leaves each point of H fixed. It is shown that M is topologically equivalent to S^3. Besides answering the question raised by Wilder, this also answers certain questions that have been raised regarding periodic transformations of S^3 into itself. In the homeomorphism of M into S^3, the "bad" points of H go into a closed 0-dimensional set W in S^3 with the following properties: (1) the complement of W is not simply connected; (2) each pair of mutually exclusive closed subsets of W can be separated by a simple surface in S^3; (3) there is a simple closed curve J in S^3-W such that each simple surface in S^3-W that separates W intersects J. (Received March 12, 1951.)

359t. R. H. Bing: *Concerning snake-like continua.*

A compact continuum M is snake-like if, for each positive number ϵ, M can be covered by an ϵ-chain. While not each compact atriodic plane continuum which does not separate the plane is snake-like, each one that contains no nondegenerate indecomposable continuum is snake-like. A nondegenerate continuum is hereditarily decomposable (hereditarily unicoherent) if each nondegenerate subcontinuum of it is decomposable (unicoherent). A necessary and sufficient condition that a compact hereditarily decomposable continuum be snake-like is that it be atriodic and hereditarily unicoherent. If M is a hereditarily decomposable snake-like continuum, M contains two points p and q such that for each positive number ϵ, an ϵ-chain from p to q covers M. (Received March 12, 1951.)

360t. M. L. Curtis and G. S. Young: *A theorem on dimension.*

This note is devoted to proving the theorem: A compact metric space X has dimension less than or equal to n if and only if there exists a light mapping of X into the n-dimensional cube. The sufficiency follows from a theorem due to Hurewicz (*Dimension theory*, Theorem VI 7) and the necessity is proved by a technique similar to that used to prove the classical imbedding theorem (Theorem 3 in *Dimension theory*). (Received March 14, 1951.)

361. Samuel Eilenberg and Saunders MacLane: *Cohomology theory of abelian groups and homotopy theory. III.*

Let X be a space with a given homotopy group $\pi=\pi_n(X)$ and all other homotopy groups trivial. It is shown that the singular homology theory of X is equivalent to the abelian homology theory of the group π, on level $n-1$. Specifically, the homology theory of X is that of a certain cell complex $K(\pi, n)$ defined algebraically in terms of the abelian group π and the integer n. For $n=1$, this complex $K(\pi, 1)=A(\pi, 1)$ has a product * of excess zero (Proc. Nat. Acad. Sci. U.S.A. vol. 36 (1950) pp. 657–663). The bar construction B (loc. cit.) applies to any complex with such a product, and yields a new complex with a new product of excess 1. B^+ is written to indicate that all dimensions are raised by 1; the new product then has excess zero. The abelian homology theory of π in level $n-1$ is that of the complex $A(\pi, n)=(B^+)^{n-1}(K(\pi, 1))$ obtained by $n-1$ applications of the bar construction. We obtain an explicit chain equivalence $A(\pi, n)\rightarrow K_a(\pi, n)$, where the subscript a denotes augmentation. The essential devices are the construction of a product in $K(\pi, n)$ which corresponds to the subdivision of the cartesian product of two simplices $\Delta_p\times\Delta_q$ into $(p+q)$-dimensional simplices, the representation of $K(\pi, n+1)$ in terms of $K(\pi, n)$ by a suitable construction W on the latter complex, and the appropriate use of the notion of generic acyclicity (loc. cit). (Received March 14, 1951.)

362. P. R. Halmos: *Normal dilations and extensions of operators.*

Let H be a subspace of a Hilbert space K, let A and B be operators on H and K respectively, and let P be the projection with domain K and range H. The operator B is called a *dilation* of A if $AP=PBP$; the operator B is called an *extension* of A if $AP=BP$. The main purposes of this paper are (1) to prove that if $|A|\leqq 1$, then A has a dilation which is unitary, (2) to prove that if $0\leqq A\leqq 1$, then A has a dilation which is a projection, and (3) to derive necessary and sufficient conditions on A in order that A have an extension which is normal. The first two results enable one to prove that if H is infinite-dimensional, then the weak closure of the set of all unitary operators is the set of all operators A such that $|A|\leqq 1$ and the weak closure of the set of all projections is the set of all operators A such that $0\leqq A\leqq 1$. The necessary and sufficient conditions for the normal extendibility of A are expressed in terms of certain matrices (generalizations of Gramians) associated with the operator A. (Received January 16, 1951.)

363*t*. Tibor Radó: *Cohomology theory for general spaces.*

Let A be a subspace of the topological space X. Cohomology groups $\tilde{H}^p(X, A)$ are assigned to the pair (X, A) as follows. A p-function c^p is an integral-valued function $c^p(x_0, \cdots, x_p)$ of $p+1$ points of X. The coboundary operator δ is defined as usual. $C^p(X, A)$ denotes the group of those p-functions which vanish locally on X and vanish on A. For $p<0$, one sets $C^p(X, A)=0$. The group $Z^p(X, A)$ is the group of those elements of $C^p(X, A)$ for which $\delta c^p=0$. The group $B^p(X, A)$ is the group of those elements of $C^p(X, A)$ which are of the form $c^p=\delta c^{p-1}$, where $c^{p-1}\in C^{p-1}(X, A)$.

Finally, $\tilde{H}^p(X, A) = Z^p(X, A)/B^p(X, A)$. Let $H^p(X, A)$ be the cohomology group employed by Spanier (Ann. of Math. vol. 49 (1948) pp. 407–427). The following results are established. If A is a nonempty closed subset of X, then $\tilde{H}^{p+1}(X, A) \approx H^p(X, A)$ for every p. For the case $A = 0$, it is shown that $\tilde{H}^{p+1}(X) \approx H^p(X)$ for $p \neq 0$, while $\tilde{H}^1(X)$ is isomorphic to the reduced 0-dimensional cohomology group. (Received March 12, 1951.)

364. E. H. Rothe: *Leray-Schauder index and Morse type numbers in Hilbert space.*

Let E be a Hilbert space and $c(x)$ a real-valued function defined for x in some neighborhood of the zero point o of E. We suppose that o is an isolated critical point of $c(x)$, that is, that $g(x) = \text{grad } c(x)$ equals o for $x = o$ and is not equal to o for all other $x \subset U$. Then the Morse type numbers m^r of dimension r ($r = 0, 1, 2, \cdots$) are well defined. If on the other hand $g(x) = x + G(x)$ where $G(x)$ is completely continuous, then also the "index" j, that is, the Leray-Schauder degree at o of $g(x)$ as map $U \rightarrow E$ is well defined. The object of the paper is to establish the relation $j = \sum_r (-1)^r m^r$ (under some additional assumptions about $g(x)$). In the special case that E is a finite-dimensional Euclidean space this relation has been proved in an earlier paper (Mathematische Nachrichten, Erhard Schmidt Festschrift, 1951). (Received March 15, 1951.)

365. M. E. Shanks: *Rings of functions on locally compact spaces.* Preliminary report.

Consider the ring $R_0(X)$ of real continuous functions with compact supports on the locally compact Tychonoff space X. Then the locally compact Tychonoff spaces X and Y are homeomorphic if and only if $R_0(X)$ and $R_0(Y)$ are isomorphic. Actually the full rings are not needed, it is necessary only that they form "Tychonoff classes" of functions. In addition X is compact if and only if $R_0(X)$ has a unit; and if a unit is adjoined, in the noncompact case, the resulting ring determines the one-point compactification of X. If X is a manifold of class C^k, let $R_k(X)$ be the ring of functions of class C^k with compact supports. Then two manifolds X and Y of class C^k are differentiably homeomorphic of class C^k if and only if $R_k(X)$ is isomorphic to $R_k(Y)$. (Received March 15, 1951.)

366. G. H. M. Thomas: *Simultaneous partitionings of two sets.*

A set M is partitionable if, for each positive number ϵ, there exists a finite collection $G = \{U_1, \cdots, U_k\}$ of disjoint, connected, open subsets of M, each of diameter less than ϵ, whose sum is dense in M. G is called an ϵ-partitioning of M. If $M_2 \subset M_1$ and G is a partitioning of M_1 such that $G' = \{U_1 M_2, \cdots, U_k M_2\}$ is a partitioning of M_2, then define G to be a simultaneous partitioning of M_1 and M_2. In this paper results are obtained regarding simultaneous partitionings of two sets, which are analogous to those obtained for partitionings by R. H. Bing (*Partitioning a set*, Bull. Amer. Math. Soc. vol. 55 (1949) pp. 1101–1110). The main theorem is: If M_1 is a compact partitionable set and M_2 is a connected partitionable closed subset of M_1, then there exists a sequence $\{G_i\}$ of simultaneous partitionings of M_1 and M_2, each a refinement of the preceding, such that each element U_j^i of G_i has property S and is of diameter less than $1/i$, and each $U_j^i M_2$ also has property S. (Received March 12, 1951.)

367t. R. L. Wilder: *The complex-like character of certain spaces.*

It is well known that the Betti numbers of dimension less than or equal to n of a compact lc^n space S are finite. Using new methods of proof, it is shown that the lc^n requirement may be replaced by the weaker assumption that S have property $(P, Q, \sim)^{n-1}$ and be n-lc. (Definitions may be found in author's Colloquium volume, *Topology of manifolds*, referred to herein as T.M.) Indeed, if a locally compact space S has property $(P, Q, \sim)^{n-1}$ and is n-lc, then it has property $(P, Q)^n$. A number of related results are obtained: For compact spaces, and $k<n$, properties lc_k^n and S_k^n are equivalent. If U is an open subset of a locally compact space, and U has property $(P, Q)^k$ while $q^{k+1}(U; x) \leqq \omega$ for all $x \in \overline{U}$, then U has property $(P, Q)^{k+1}$. (Compare Theorem XI 6.8, T.M.) A nasc that an n-dimensional, locally compact space S should have property $(P, Q)_k$ and be lc_k^n, $k \leqq n$, is that $p_r(x) \leqq \omega$ for all $x \in S$ and $r=k$, $k+1, \cdots, n$ (see Theorem VII 2.25, T.M.). Various other results are obtained, such as generalizations of Theorems VI 5.1, VII 2.26, Theorems 1.3, 2.1, 2.3, 3.6, 3.10, 3.11 of Chapter XI of T.M., and on the decompositions of spaces into "prime parts" (see Amer. J. Math. vol. 63 (1941) pp. 691–697). (Received March 13, 1951.)

368. Oswald Wyler: *Order and topology in a projective plane.*

In a projective plane satisfying the axioms of incidence and of order, a topology of any line is derived from its order. Projectivities are then homeomorphisms. If l is a line, a point-set of the plane is said to be *l-convex* if it contains no point of l, and if its intersection with any line is either void or a point or a segment. Taking sets l-convex with respect to some line l and containing three points not on a line as a defining system of neighborhoods, the plane of points is seen to be a regular topological space. A line is a closed set, and its relative topology is the order-topology. Dually, a topology of the plane of lines is defined. It is shown that the line joining two distinct points is a continuous function of these points, and that the point of intersection of two lines is a continuous function of the lines. No configuration theorem is needed. If the lines are homeomorphic to the real projective line, the plane will be homeomorphic to the real projective plane. An example of such a plane is given, in which the Theorem of Desargues does not hold. (Received February 23, 1951.)

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