algebra form, under set-union, a cardinal algebra in which $a+a=a$.

Another interesting feature of the book is the construction, given a cardinal algebra $\mathfrak{A}$ and a group G of "partial automorphisms" of $\mathfrak{A}$, of a "refinement algebra," very similar to a cardinal algebra, in which elements of $\mathfrak{A}$ equivalent under G (directly or by decomposition) are identified. In this direction, various abstract analogs of theorems relating to the existence of measure, and to the Banach-Tarski paradox, are proved. (Let $\mathfrak{A}$ consist of Borel sets, and let G be the group of isometries of space.) The algebra of cardinal numbers under addition is deduced from the algebra of sets by the same construction.

Another section deals with the relation between "cardinal algebras" and other types of algebraic systems—especially semigroups and distributive lattices. Finally, an appendix discusses "cardinal products of isomorphism types"—that is, with the direct factorization of general algebraic systems with a binary operation and a zero. The results here are related to the monograph *Direct decompositions of finite algebraic systems*, by the author and Bjarni Jónsson.

It seems certain, to the reviewer, that the postulates and ingenious deductions of Professor Tarski will permanently enrich modern algebra—at the same time that they show once more the value of considering infinitary operations. On the other hand, it seems less clear that the particular combinations of conditions labelled by the author "cardinal algebra," "generalized cardinal algebra," and "refinement algebra" will survive without modification. At all events, the author is to be congratulated for penetrating deep into new and heretofore uncharted mathematical territory; the book is a "must" for everyone seriously interested in modern algebra or set theory.

GARRETT BIRKHOFF

Theory of functions. By J. F. Ritt. Rev. ed. New York, Kings Crown Press, 1947. 10+181 pp. $3.00.

A student's introduction to the theory of functions has certain aspects in common with a youth's emergence into the adult world. Here he meets directness and subtlety, power and simplicity, beauty and rigor in quantity and proportion not previously experienced. A mathematician who undertakes to mediate this introduction by means of a book must be in control of these properties. Beyond that he needs the tact which prevents him from overwhelming the student with needless detail or deluding him with insufficient mathematical content. In the book under review these conditions have been met with ease. The teacher in search of a textbook will find, agreeably, that each chapter of the book has one theme and may be presented in one lecture.

The book seems to fall into five parts:

1. Real numbers and real functions occupy nine chapters covering 51 pages. The real numbers are identified with the non-terminating decimals; the arithmetic for terminating decimals being extended to them by means of the least upper bound theorem. There follow theorems on limits and on linear point sets. For real functions there are considerations of continuity, uniformity, monotoneity, and the basic theorems of the differential and integral calculus. This portion concludes with a study of the convergence, integrability, and differentiability of infinite series and the exhibition of an integrable function which is discontinuous on a dense set.

2. Functions of two variables, regions and curves in the plane, and curvilinear integrals are discussed in the next 40 pages. An interesting feature here is the distinction between the equivalence and inverse equivalence of two curves according as one is obtained from the other by an increasing or decreasing transformation of the parameter interval. This is neatly exploited in orientation questions involved in curvilinear integrals. In these pages are introduced the complex numbers, complex functions and their differentiation, the Cauchy-Riemann equations, and the conformal mapping property of analytic functions.

3. The next 26 pages deal with the Cauchy integral theorem and formula. This material presents a most serious problem of exposition to anyone writing on analytic functions. A treatment of this problem on the clean mathematical level set for himself by the author in this work would have led to a digression disrupting the pattern of the book. It is his choice to give an elegant discussion of the topology of the triangle and refer to the literature for the extension to Jordan curves. This manner of dealing with the difficulty offers to the lecturer using the text the inviting opportunity to expound his favorite proof of the Jordan curve theorem, a matter on which there is a wide range of taste.

4. With the machinery developed up to this point the analytic product is now obtained in 43 pages covering the series of Taylor and Laurent, the fundamental theorem of algebra as a corollary to Liouville's theorem, the zeros and poles of analytic functions. There is a treatment of the classical elementary functions, periodicity, and infinite products.

5. The concluding 25 pages expound such results as the Weierstrass factorization theorem, the Mittag-Leffler theorem on mero-

morphic functions, the neighborhood preserving property of analytic functions as a corollary to Rouché's theorem, the theory of residues. The concept of multiple valued function is prepared for by a neat discussion of the variation in amplitude of a function along a curve. The Riemann surface is mentioned and there is a concluding chapter on analytic continuation.

If we consider a presentation of function theory which is elegant without being formidable and simple without being trivial, then we have in Professor Ritt's *Theory of functions* a simple and elegant book.

L. W. COHEN