THE FEBRUARY MEETING IN CHICAGO

The four hundred forty-fourth meeting of the American Mathematical Society was held at the University of Chicago, on Saturday February 26, 1947. The attendance was eighty-one, including the following seventy-seven members of the Society:

A. A. Albert, W. R. Allen, Reinhold Baer, E. W. Banhagel, S. F. Bibb, W. M. Boothby, A. J. Brandt, C. R. Cassity, Herman Chernoff, E. W. Chittenden, A. M. Christiansen, H. M. Clark, Byron Cosby, D. M. DeWitt, Flora Dinkines, John Dyer-Bennet, W. F. Eberlein, Chester Feldman, J. V. Finch, Harley Flanders, L. R. Ford, Evelyn Frank, C. G. Fry, M. P. Gaffney, David Gilbarg, S. H. Gould, L. M. Graves, L. W. Griffiths, William Gustin, M. M. Gutterman, P. R. Halmos, H. L. Harter, I. N. Herstein, L. A. Hostinsky, Ralph Hull, M. M. Johnsen, Louis Joseph, D. E. Kibbey, D. D. Kosambi, M. S. Kramer, W. C. Krathwohl, M. Z. Krzywoblocki, E. P. Lane, J. S. Leech, A. W. McGaughey, Saunders MacLane, Morris Marden, J. M. Mitchell, E. J. Moulton, S. B. Myers, Leopoldo Nachbin, Zeev Nehari, E. H. Ostrow, D. W. Pounder, W. T. Reid, Haim Reingold, Herman Rubin, L. J. Savage, A. C. Schaeffer, Lowell Schoenfeld, W. T. Scott, I. E. Segal, M. A. Seybold, H. A. Simmons, Annette Sinclair, A. H. Smith, E. H. Spanier, M. P. Steele, A. F. Strehler, E. A. Trabant, E. F. Trombley, Henry Van Engen, L. R. Wilcox, R. E. Wright, L. C. Young, J. W. T. Youngs, Antoni Zygmund.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, Professor S. B. Myers delivered an address entitled *Normed linear spaces of continuous functions*. Professor Myers was introduced by Professor L. M. Graves of the University of Chicago. Contributed papers were presented to the Society at a session on Saturday afternoon, presided over by Professor L. R. Ford of the Illinois Institute of Technology.

Visiting mathematicians and their guests were entertained by Mrs. Saunders MacLane on Saturday afternoon following the conclusion of the session for contributed papers.

Abstracts of all papers presented at the meeting are given below. Papers read by title are indicated by the letter "*t*."

ALGEBRA AND THEORY OF NUMBERS

263*t*. Harvey Cohn: *Minkowski's conjectures on critical lattices in the metric* $(|\xi|^p+|\eta|^p)^{1/p}$.

The conjecture deals with the minimum constant c_p for which each lattice in the $\xi\eta$-plane will necessarily have two points no further apart than $c_pA^{1/2}$ in the sense of the metric $(|\xi|^p+|\eta|^p)^{1/p}$, $p>1$, where A is the eculidean area of the fundamental parallelogram of the lattice. The main result is that for p large enough, $c_p=2^{1/p}(1-\tau_p)^{1/2}/(1+\tau_p)^{1/2}$, where τ_p is the root of $\tau_p^p+1=2(1-\tau_p)^p$. Its significance lies in the fact that it verifies, in part, some conjectures of Minkowski, other parts of which have been recently disproved. The solution is determined, in principle, by a theorem of Minkowski which in this case asserts that $c_p=\delta_p^{-1/2}$, where δ_p is the minimum area of the simple infinity of parallelograms one vertex of which lies on the origin and the other three vertices of which lie of the curve $|\xi|^p+|\eta|^p=1$. The conditions defining minimum are treated asymptotically as p approaches infinity, and it is shown that for p large enough, only the obvious symmetry positions enter into consideration as optima. (Received January 10, 1949.)

264. I. N. Herstein: *A proof of a conjecture of Vandiver.*

Vandiver, in Ann. of Math. vol. 48 (1946) p. 28, conjectured that the Wedderburn theorem, that every finite skew-field is commutative, could be generalized as follows: Every finite, non-commutative ring has an element which is a divisor of zero and is not in the centrum of the ring. Using elementary ring theory, this conjecture is proved. (Received January 21, 1949.)

265t. C. M. Price: *Jordan division algebras.* Preliminary report.

It is shown that the only central Jordan division algebras $S^{(+)}$ over an algebraic number field F which are not fields are obtained by replacing ordinary multiplication by the quasi-multiplication $a \bigcirc b=(ab+ba)/2$ in S, where S is either (1) an associative central division algebra D of odd degree over F or (2) the set of all J-symmetric elements of D of odd degree over F, where J is of the second kind. More generally if S is an associative central simple crossed product of degree n over any field F, then $S^{(+)}$ is a Jordan division algebra if and only if S is a division algebra and n is odd. If S is the set of all J-symmetric elements of an associative central simple algebra A of degree n over its center F, then $S^{(+)}$ or $A^{(+)}$ is a Jordan division algebra not a field only if A is a division algebra. In this case if J is of the first kind so that $n=2^m$, and A contains an imprimitive quartic field over F when $n>2$, then $S^{(+)}$ is a Jordan division algebra if and only if $S^{(+)}$ is a field and $n=1, 2$. Also if n is odd and B is any subset of A (division algebra), then $B^{(+)}$ can have no (Jordan) divisors of zero. (Received January 19, 1949.)

266t. K. G. Ramanathan: *On the Fourier coefficients of certain modular forms.*

The present paper is devoted to the study of the congruence properties of the arithmetic function $P_\nu(n)$ defined by $\nu>0$, $\sum_0^\infty P_\nu(n)x^n=[(1-x)(1-x^2)\cdots]^{-\nu}$ for the moduli 5^α and 7^β. One of the many results proved is: if $\nu>0$, $\nu\equiv -24 \pmod{420}$ and $24m\equiv\nu \pmod{5^\alpha 7^\beta}$ then $P_\nu(m)\equiv 0 \pmod{5^\alpha 7^\beta}$. The Ramanujan-Watson congruences for the partition function are contained as particular cases. (Received January 13, 1949.)

267. H. A. Simmons: *Interpretation of belonging-to exponents in terms of ever-repeating decimals.*

In this paper, some number scales used have positive bases; others, negative bases. When the base of a scale that is used is negative, the digits in the associated decimal may not all be taken with the same sign. When they are not, the decimal is called a "pseudo-decimal." In a particular situation, our procedure leads naturally either to a customary decimal or to a "pseudo-decimal." Our basic result is this: *if* $(a', m)=1$, *where* $m>1$ *and* a' *is numerically larger than* 1, *then a necessary and sufficient condition that* a' *belong to the exponent* e *modulo* m *is that* $1/m$ *be expressible in the scale of* a' *by an ever-repeating decimal of shortest period* e. One corollary of this theorem is about the expression of b/m as an ever-repeating decimal when $(b, m)=1$,

$m>1$, and $1 \leqq |b| < m$; another corollary relates to the replacement of a base a' by bases of the form $a'+km$; and finally five additional theorems are stated which are analogous to known theorems of elementary number theory. (Received March 2, 1949.)

268t. M. F. Smiley: *A remark on a theorem of Marshall Hall.*

The identity $x(yz-zy)^2=(yz-zy)^2x$ characterizes Cayley-Dickson algebras among alternative but not associative division rings. (Cf. Marshall Hall, *Projective planes*, Trans. Amer. Math. Soc. vol. 54 (1943) pp. 229–277.) This follows from Hall's proof and a result of A. A. Albert (*Absolute valued algebraic rings*, Bull. Amer. Math. Soc. Abstract 54-11-416). A geometric consequence is that the stated identity and Hall's Theorem L ensure the uniqueness of the coordinate ring in a projective plane. (Received January 8, 1949.)

ANALYSIS

269. William Gustin: *Areal mean value families.*

A family $\mathfrak{S}$ of plane surfaces is called an areal mean value family if every function f regular in a simply-connected domain D has the same areal mean value over any two surfaces of $\mathfrak{S}$ covered by D. Many areal mean value families besides the concentric circles (Gauss) and the confocal ellipses (Asgeirsson) are shown to exist. (Received January 12, 1949.)

270t. Samuel Karlin: *Orthogonal properties of independent functions.*

This paper presents a systematic study of the analytical orthogonal properties of independent functions. The first part treats of convergence of expansions of functions in series of independent functions. It is shown under mild conditions that almost all modes of convergence are equivalent. In the second part, the relationship of convergence and summability questions are investigated. The Lebesgue kernel of an independent system is shown to be nonsummable. Finally, the connection of independent systems and lacunary orthogonal systems is discussed. (Received February 6, 1949.)

271. Josephine M. Mitchell: *On convergence and (C, 1, 1) summability of double orthogonal series.*

Let (i) $\sum_{m,n=1}^{\infty} a_{mn}\phi_{mn}(P)$, $a_{mn}=\int_E f(P)\phi_{mn}(P)dA_P$, be the orthogonal development of an arbitrary function $f(P)$ of class L^2 defined on a measurable set E of r-dimensional Euclidean space, with respect to the complete orthonormal system $\{\phi_{mn}(P)\}$ $(m, n=1, 2, \cdots)$ of functions of class L^2. The rôle of the Lebesgue functions, $L_{mn}(P)$ $=\int_E |K_{mn}(P, Q)| dA_Q$, where $K_{mn}(P, Q)$ is the mnth kernelfunction, $\sum_{i,j=1}^{m,n}\phi_{ij}(P)\phi_{ij}(Q)$, in the convergence of series (i) is investigated. It is proved that if $L_{mn}^*(P)$ $=\int_E \max_{1\leqq j\leqq n} |K_{mj}(P, Q)| dA_Q$ is $O(u^2(m))$ $(u(m)\leqq u(m+1))$, then the sequence $s_{mn}(P)/u(m)$, where $s_{mn}(P)$ is the mnth partial sum of (i), approaches a limit amost everywhere in E. In particular, if E is the Cartesian product of two measurable sets E_1 and E_2 and $\{\phi_{mn}\}\equiv\{\phi_m^{(1)}\phi_n^{(2)}\}$, where $\{\phi_n^{(\kappa)}\}$ is a complete orthonormal set defined on E_κ $(\kappa=1, 2)$, then the result corresponding to the above for simple orthogonal series may be exactly generalized (S. Kaczmarz, Studia Math. vol. 1 (1929) p. 101) in the case of positive kernel functions. Similar results are obtained for (C, 1, 1) summability. (Received January 13, 1949.)

272t. Zeev Nehari: *A class of domain functions and some allied extremal problems.*

Let D be a smoothly bounded finite domain of connectivity n in the complex z-plane, s the length-parameter on the boundary Γ of D, and $\lambda(s)$ a continuous positive function on Γ. It is shown that there always exist two uniquely determined single-valued analytic functions $K_\lambda(z, \zeta)$ and $L_\lambda(z, \zeta)$ which are regular in D with the exception of a simple pole of $L_\lambda(z, \zeta)$ at $z=\zeta$ ($\zeta \in D$) and which, for $z \in \Gamma$, are connected by the identity $\lambda(s)\overline{K_\lambda(z, \zeta)}ds = -iL_\lambda(z, \zeta)dz$. This identity implies that $K_\lambda(z, \zeta)$ has the reproducing property $\int_\Gamma \lambda(s)\overline{K_\lambda(z, \zeta)}f(z)ds = f(\zeta)$ with respect to the class C of functions $f(z)$ which are regular and single-valued in D and whose boundary values are of class $L^2(\Gamma)$. The function $K_\lambda(z, \zeta)$ can be computed by means of the bilinear formula $K_\lambda(z, \zeta) = \sum_{\nu=1}^{\infty} \phi_\nu(z)\overline{\phi_\nu(\zeta)}$, where $\{\phi_\nu(z)\}$ is a complete set of functions of C orthonormalized by the conditions $\int_\Gamma \lambda(s)\phi_\nu(z)\overline{\phi_\mu(z)}ds = \delta_{\nu\mu}$. An algorithm of similar type yields the function $L_\lambda(z, \zeta)$. It is then shown that the functions $K_\lambda(z, \zeta)$, $L_\lambda(z, \zeta)$, and certain combinations of them provide the solutions for a number of extremal problems in the theory of conformal mapping. (Received January 12, 1949.)

273. Lowell Schoenfeld: *On second order linear partial differential equations with singular coefficients.*

Let $L(U) = U_{z\bar{z}} + AU_z + BU_{\bar{z}} + CU$ where A, B and C are functions of the two independent complex variables $z = x+iy$, $\bar{z} = x-iy$ and x and y can take complex values. Using the work of Bergman (Rec. Math. (Mat. Sbornik) N.S. vol. 2 (1937) pp. 1169–1198 and Trans. Amer. Math. Soc. vol. 57 (1945) pp. 299–331), the author proves that if $L(U) = 0$ and certain auxiliary conditions are satisfied then $U(z, \bar{z})$ is singular in the plane $\bar{z} = \bar{z}_0$. The following is typical of the two principal results. If $k \geqq 0$ is an integer, $B_{\bar{z}\bar{z}} - A_{zz} = 0 = 2^{-1}(2k-1)A_z - 2^{-1}(2k+1)B_{\bar{z}} + C - AB$, $B(z, \bar{z})$ is singular at $\bar{z}_0 \neq 0$ and has no other singularity, $A(z, \bar{z})$ is entire, H_{mn}, defined by $U \exp \{\int_0^{\bar{z}} A(z, \zeta)d\zeta + \int_0^z B(\xi, 0)d\xi\} = \sum_{m,n=0}^{\infty} H_{mn} z^m \bar{z}^n$, is such that $H_{0n} = 0$ for $n \geqq 1$, $H_{m0} = 0$ for $m < k$, $H_{k0} \neq 0$, $H_{m+1,0}/H_{m,0} \to 0$ as $m \to \infty$ and $L(U) = 0$, then $U(z, \bar{z})$ is single-valued for $|z|$, $|\bar{z}| < \infty$, is singular in the plane $\bar{z} = \bar{z}_0$, and has no other singularities for $|z|$, $|\bar{z}| < \infty$. (Received January 12, 1949.)

Applied Mathematics

274. David Gilbarg: *A characterization of non-isentropic irrotational flows.*

Steady irrotational flows are considered for which entropy is required to be constant only along streamlines. It is shown that all plane subsonic irrotational flows are in the large either isentropic or are vortex flows. If certain general restrictions are placed on the boundaries of the flow region, the same result holds for mixed and supersonic flows. Analogous theorems are proved for axially symmetric and general spatial flows. The proofs depend on uniqueness theorems for the Cauchy initial value problem applied to the equation for the velocity potential of isentropic irrotational flow. A new uniqueness theorem is proved for the case of parabolic data on the initial curve, that is, for data on the sonic line. (Received January 14, 1949.)

275. M. Z. Krzywoblocki: *On shock theory in viscous gases.*

In most existing shock theories the assumption is that the shock curve is a geometric surface (line) of discontinuity in pressure, velocity, and so on. The location of the shock and its shape is more or less unknown at the very beginning of the problem. The author, starting with the application of the methods known from the theory of the "limiting lines" in an inviscid fluid to a viscous fluid, has obtained new results. The first result is that the "limiting lines" cannot exist in a viscous fluid. The question of whether a limiting line is a characteristic feature of a transformation (hodograph transformation) may be solved by using other kinds of transformation. The second result is that the curve of the beginning of a shock is the locus of points in which the change of pressure with respect to the running coordinate along a streamline is of the same order as the change of the components of the stress tensor with respect to the same coordinate. Hence the criterion for the beginning of the shock in a viscous fluid is the limited deformation in a continuous medium. (Received January 14, 1949.)

Geometry

276t. Edward Kasner and John DeCicco: *Physical families in conservative fields of force.*

A physical system S_k of ∞^3 curves in a conservative field of force is obtained as the extremals of the variation problem $\int(W+h)^{(k+1)/2}ds=\min$, where W is the work function and h is the energy constant. According as k is 0, 1, -2, the system S_k consists of dynamical trajectories, general catenaries, brachistochrones. A velocity system is defined by the variation problem $\int e^{W/v_0^2}ds=\min$. In a non-homothetic conformal representation of a surface Σ_1 upon a surface Σ, complete systems S_k do not, in general, correspond. There is only one conservative field of force, namely the one for which the work function is $w=(ds_1/ds)^{2/(k+1)}$, on Σ, for which the complete system S_k becomes a complete system S_k on Σ_1. The corresponding work function on Σ_1 is $w_1=1/W$. There is no complete system S_k on Σ, which by a non-homothetic conformal map on Σ_1 becomes a complete system S_{k_1} with $k_1\neq k$ on Σ_1. (Received December 30, 1949.)

277t. Edward Kasner and John DeCicco: *Physical systems of curves in space.*

Connected with an arbitrary positional field of force in space are the systems S_k of ∞^5 curves along any one of which a constrained motion is possible such that the osculating plane at each point contains the force vector F, and the pressure P is proportional to the normal component N of F. Thus $P=kN$. Special cases are (a) dynamical trajectories, (b) general catenaries, (c) generalized brachistochrones, (d) velocity curves, according as k is (a) 0, (b) 1, (c) -2, (d) ∞. The locus of the centers of the osculating spheres of the curves of a system S_k through a given lineal element is a straight line. By varying the direction through the point, the associated lines form a congruence Ω of order one and class three. This congruence Ω is composed of the secants of a twisted cubic curve Γ. The associated quadrics through Γ are studied, from which can be deduced a geometric interpretation of the curl of F. Many other related results are also discussed. (Received December 27, 1949.)

Logic and Foundations

278t. A. R. Schweitzer: *On the place of Kant's Critique of Pure Reason in a genetic logic of mathematics.*

In his article, *Les idées directrices de la logique génétique des mathématiques* (Rev. de Mét. et de Mor., Paris, 1914) the author quotes from an article by Felix Klein mediating, in effect, between intuition and logical deduction in evolutionary mathematics. In this paper Klein's remark is interpreted as a special case of a statement by Kant (*Critique of pure reason*, translated by F. Max Müller, New York, 1911, p. 41): " . . . it is equally necessary to make our concepts sensuous . . . as to make our intuitions intelligible . . . ." This leads to finding a place for Kant's Critique in a genetic logic of mathematics by interpreting the Critique as a theory of knowledge elaborating a relation between intuition and logic. For intuition is associated by Kant with sensibility and logic is subdividied by him into analytic, associated with the understanding, and dialectic, associated with reason. Also Kant assumes (ibid., p. 242; compare p. 563) that "All our knowledge begins with the senses, proceeds thence to the understanding, and ends with reason." (Received January 13, 1949.)

279*t*. A. R. Schweitzer: *Outline of a theory of knowledge suggested by mathematics*. Preliminary report.

This paper consists of three parts: I. Systems of notation (representation); II. Terms and processes. III. Relations. Instruments of notation are: letters, words, signs, symbols. Systems of notation include: language, literature, philosophy, science and mathematics. Terms include: entities, things, objects, facts, thoughts, concepts, ideas, elements, points, units, monads, atoms. Processes include: classification, orientation, working hypothesis, assumption, definition, composition, inference, method, proof, existence, construction, approximation, limitation, replacement, transformation. Mathematics is held to be a science of relations. In part II reference is made to Kant's statement, "Whatever the process and the means may be by which knowledge reaches its objects, there is one that reaches them directly, and forms the ultimate material of all thought, viz. intuition . . . ." (*Critique of pure reason*, translated by F. Max Müller, New York, 1911, p. 15). (Received January 13, 1949.)

Topology

280*t*. M. K. Fort: *Essential and non-essential fixed points*.

Let X be a compact metric space having the fixed point property. Let $f\in X^X$ and p be a fixed point under f. The point p is an essential fixed point under f if corresponding to each neighborhood U of p there exists $\epsilon>0$ such that every g for which $\rho(f, g)<\epsilon$ has a fixed point in U. An approximation theorem is proved which shows that every $f\in X^X$ can be approximated arbitrarily close by a transformation, all of whose fixed points are essential. It is proved that if f has a single fixed point, then this fixed point is essential. Another existence theorem is proved, to the effect that if X is a n-manifold and the set of fixed points of $f\in X^X$ is totally disconnected, then f has at least one essential fixed point. Various examples and special cases are discussed. (Received January 13, 1949.)

281*t*. E. E. Moise: *A note on the pseudo-arc*.

It has been shown by R. H. Bing (Duke Math. J. vol. 15 (1948) pp. 729–742) that a certain continuum described by the author (Trans. Amer. Math. Soc. vol. 63 (1948) pp. 581–594) is homogeneous. The present note gives a proof of Bing's result, based on the methods of the author's original paper. (Received February 3, 1949.)

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