THE NOVEMBER MEETING IN PASADENA

The four hundred thirtieth meeting of the American Mathematical Society was held at the California Institute of Technology, Pasadena, California, on Saturday, November 29, 1947. The attendance was approximately 115, including the following 82 members of the Society:

O. W. Albert, H. L. Alder, R. F. Arens, S. P. Avann, L. H. Bechtolsheim, E. F. Beckenbach, H. W. Becker, M. M. Beenken, Clifford Bell, E. T. Bell, H. F. Bohnenblust, Herbert Busemann, Ralph Byrne, W. D. Cairns, F. H. Campbell, L. H. Chin, F. M. Clarke, L. M. Coffin, F. E. Cothran, E. L. Crow, D. R. Curtiss, P. H. Daus, Harold Davenport, R. W. Davies, R. P. Dilworth, L. K. Durst, Arthur Erdélyi, G. C. Evans, G. E. Forsythe, W. H. Glenn, E. C. Goldsworthy, J. W. Green, Leonard Greenstone, H. J. Hamilton, W. L. Hart, Olaf Helmer, J. G. Herriot, M. R. Hestenes, P. G. Hoel, Alfred Horn, D. H. Hyers, Rufus Isaacs, C. G. Jaeger, P. B. Johnson, P. J. Kelly, Cornelius Lanczos, D. H. Lehmer, Harold Luxenberg, S. L. McDonald, G. F. McEwen, Rhoda Manning, A. V. Martin, A. D. Michal, E. D. Miller, T. E. Oberbeck, J. W. Odle, L. J. Paige, R. S. Phillips, W. T. Puckett, W. C. Randels, H. P. Robertson, J. B. Robinson, R. M. Robinson, C. H. Savit, Henry Scheffé, G. E. F. Sherwood, Ernst Snapper, I. S. Sokolnikoff, R. H. Sorgenfrey, D. V. Steed, A. C. Sugar, J. D. Swift, Alfred Tarski, E. B. Tolstead, C. A. Truesdell, S. E. Urner, F. A. Valentine, Morgan Ward, L. E. Wear, P. A. White, R. M. Winger.

The morning session was devoted to research papers and to the hour address, *Electronic aids to the theory of numbers*, by Professor D. H. Lehmer of the University of California. Professor Morgan Ward presided. Additional research papers were presented in the afternoon in two sections, analysis and algebra, at which Professors E. F. Beckenbach and E. T. Bell, respectively, presided.

Following the meetings, tea was served at the Athenaeum through the hospitality of the Department of Mathematics of the California Institute of Technology.

Abstracts of papers read at the meeting follow below. Abstracts whose numbers are followed by the letter "*t*" were presented by title. Dr. Karlin was introduced by Professor Morgan Ward. Paper number 87 was read by Dr. Horn, paper 88 by Professor Tarski, and paper 90 by Miss Chin.

ALGEBRA AND THEORY OF NUMBERS

85. H. L. Alder: *The nonexistence of certain identities in the theory of partitions and compositions.*

Let $q_{d,m}(n)$ be the number of partitions of n into parts differing by at least d, each part being greater than or equal to m. Identities involving $q_{1,1}(n)$ (Euler's identity) and $q_{2,1}(n)$ and $q_{2,2}(n)$ (Rogers-Ramanujan identities) are known. It is shown in this paper that, aside from a simple generalization of Euler's identity, no other such identities can exist. The following two theorems, both of which were proved for the case $m=1$ by D. H. Lehmer (Bull. Amer. Math. Soc. vol. 52 (1946) pp. 538–544), are proved: The number $q_{d,m}(n)$ of partitions of n is not equal to the number of partitions of n into parts taken from any set of integers whatsoever unless $d=1$, or $d=2$ and $m=1, 2$. The number $q_{d,m}(n)$ of partitions of n is not equal to the number of partitions of n into distinct parts taken from any set of integers whatsoever unless $d=1$. The nonexistence of certain generalizations of an identity of I. Schur involving partitions into parts differing by 3 or more is also proved. Furthermore it is shown that the results above can be extended to compositions. (Received October 13, 1947.)

86. S. P. Avann: *Ternary distributive semi-lattices.*

A ternary distributive semi-lattice $\mathfrak{J}$ is a set of elements closed with respect to a ternary operation satisfying (T1) $(a, a, b)=a$, (T2) (a, b, c) symmetric in all three letters, (T3) $((a, b, c), d, e)=((a, d, e), b, (c, d, e))$. Representing the elements of $\mathfrak{J}$ by points and joining elements b and c by a "link" of length 1 whenever $(b, x, c)=b$ or c for all x, one obtains the graph: $(\mathfrak{J})$ $\mathcal{G}$ of $\mathfrak{J}$. Ternary distance $\rho[x, a]+\rho[x, b]+\rho[x, c]$ in $\mathcal{G}(\mathfrak{J})$ has a unique minimum for $x=(a, b, c)$. Conversely, if to each unordered triple a, b, c of elements of a connected graph $\mathcal{G}$ there corresponds an element $[a, b, c]$ having unique minimal ternary distance, then $\mathcal{G}$ is a ternary distributive semi-lattice with $(a, b, c)=[a, b, c]$. For each $a\in\mathfrak{J}$ a partially ordered set $\rho(a, \mathfrak{J})$ may be defined whose ascending chains are the minimal chains joining a to the other elements of $\mathfrak{J}$. Each $\rho(a, \mathfrak{J})$ is an ideal of a distributive lattice of rank equal to the number n, independent of a, of join irreducibles of $\rho(a, \mathfrak{J})$. If a has a complement a' satisfying (T4) $(a, x, a')=x$ for all x, then $\rho(a, \mathfrak{J})$ is a distributive lattice. (Received November 20, 1947.)

87. Alfred Horn and Alfred Tarski: *Measures in Boolean algebras.*

A non-negative functional m on a Boolean algebra B is called a *measure on* B if $m(x+y)=m(x)+m(y)$ for any disjoint $x, y\in B$, and $m(1)=1$; it is called *strictly positive* if $m(x)=0\rightarrow x=0$, and *countably additive* if $m(\sum_{n<\infty}x_n)=\sum_{n<\infty}m(x_n)$ for any disjoint $x_n\in B$. The definition of measure, in one of its equivalent forms, is applied to an arbitrary set $S\subseteq B$; every measure m on S is shown to be extendible to a measure m' on B (m' being two-valued if m is two-valued). Furthermore, various conditions for the existence and non-existence of strictly positive or countably additive measures are established. For example, if B has a strictly positive measure, every uncountable $S\subseteq B$ includes an uncountable subset T such that $x, y\in T\rightarrow x\cdot y\neq 0$. Also, if a countable set $D\subseteq B$ is "dense" in B (that is, if every $x\in B$ is a sum of elements in D), then (i) B is isomorphic with a field of sets of integers; (ii) B has a strictly positive measure which is not countably additive; (iii) B has a countably additive measure only if it is not atomless; (iv) B has a strictly positive countably additive measure only if it is atomistic. (Received October 22, 1947.)

88. Bjarni Jónsson and Alfred Tarski: *Boolean algebras with operators.*

A function F mapping a Boolean algebra B onto a set $C\subseteq B$ is called an *operator* if $F(x+y)=F(x)+F(y)$ for $x, y\in B$; F is *complete* if $F(\sum x_i)=\sum F(x_i)$ for $x_i\in B$ (whenever $\sum x_i\neq 0$ exists), and *normal* if $F(0)=0$. A function of many arguments is called an operator if it is an operator in each argument; similarly for complete or normal operators. Closure algebras, relation algebras, projective algebras are Boolean algebras with operators (see McKinsey-Tarski, Ann. of Math. vol. 45, and the following two abstracts). *Extension Theorem*: (i) Every algebra B with operators $F_0, F_1, \cdots$, can be extended to a complete atomistic algebra B' with complete operators $F_0', F_1', \cdots$; (ii) every identity in B involving Boolean addition, multiplication, and operators F_i becomes an identity in B' when F_i are replaced by F_i'. Given an n-termed relation (a set of n-termed sequences) R and sets $X_1, \cdots, X_{n-1}$, let $R^*(X_1, \cdots, X_{n-1})$ be the set of all y's with $(x_1, \cdots, x_{n-1}, y) \in R$ for some $x_i \in X_i$. *Representation Theorem*: Every algebra with normal operators is isomorphic to a field of sets with operators of the form R^*. Example: every closure algebra is isomorphic to a field of sets with the closure operator R^* where R is a reflexive and transitive binary relation. (Received October 21, 1947.)

89*t*. Bjarni Jónsson and Alfred Tarski: *Representation problems for relation algebras.*

A *relation algebra* (RA) A is a Boolean algebra with a binary associative operator $|$ and a unary operator $\smile$ such that: $|$ has the unit element e; $a^{\smile\smile}=a$; $(a|b)^{\smile}=b^{\smile}|a^{\smile}$; $a|\overline{a|b} \leqq \bar{b}$. (See Tarski, Journal of Symbolic Logic vol. 6, and Abstract 88.) Examples: I. *Proper relation algebras* (PRA's)—families A' of subrelations of a binary relation U closed under set addition $R+S$, complementation $U-R$, relative multiplication $R|S$, conversion $R^{\smile}$; the set of all (a, a) with $a \in U$ is in A'. II (by McKinsey). *Frobenius algebras* (FA's)—families A'' of subsets of a group G closed under set addition and complementation, complex multiplication and inversion; the smallest subgroup of G is in A''. Results: 1. A is extendible to a complete atomistic RA. 2. A is isomorphic with an algebra where $R+S$, $R|S$, $R^{\smile}$ (but not complementation) have the meaning in I. 3. An atomistic A where $a^{\smile}|a \leqq e$ (or $a^{\smile}|a=e$) for every atom a is isomorphic with a PRA (or FA). 4 (by McKinsey-Tarski). A is simple if and only if $a \neq 0 \rightarrow 1|a|1=1$. 5. A PRA which is simple is isomorphic with a PRA A' where U is a Cartesian set-square; and conversely. Problems: Are all RA's isomorphic with PRA's? Are all RA's with $a|b=0 \rightarrow (a=0 \vee b=0)$ isomorphic with FA's? (Received October 21, 1947.)

90. Louise H. Chin and Alfred Tarski: *Remarks on projective algebras.*

By Everett-Ulam, Amer. J. Math. vol. 68, projective algebras (PA's) are Boolean algebras with additional operations $O_1(a)=a_x$, $O_2(a)=a_y$ satisfying certain postulates (some rather involved). Example: proper PA's—non-empty families of subsets of a Cartesian product $X \times Y$ closed under set addition, $A+B$; complementation, $(X \times Y)-A$; projections on X-axis, A_x, and Y-axis, A_y. By definition, in any PA: $a^x=$ (the largest b with $b_x=a_x$); similarly a^y; $p=1_{xy}$. A PA can now be characterized as a Boolean algebra with operations a^x, a^y, and a distinguished element p satisfying mutually independent postulates: $0^x=0$; $0^y=0$; $a \leqq a^x$; $a \leqq a^y$; $(a^x \cdot b)^x=a^x \cdot b^x$; $(a^y \cdot b)^y=a^y \cdot b^y$; $a \neq 0 \rightarrow a^{xy}=1$; p is atom; $p=p^x p^y$. One redefines a_x as $a^x \cdot p^y$; similarly a_y. The two postulate systems for PA's prove equivalent. By Abstract 89 above, a^x, a^y being operators, every PA is extendible to a complete atomistic *PA* (result obtained differently by McKinsey, Bull. Amer. Math. Soc. Abstract 52-7-227); consequently, by Everett-Ulam, it is isomorphic with a proper PA. Both results extend to related algebras, for example, Boolean algebras with operations a^x, a^y satisfying all postulates above not involving p; PA's supplemented by an operator a^c with

$a^{cc}=a$, $a^{xc}=a^{cv}$. (In a proper PA, A^c is obtained from A by interchanging coordinates in each point.) PA's are closely related to RA's of Abstract 89. (Received October 21, 1947.)

91*t*. C. A. Rogers: *The signatures of the errors of simultaneous Diophantine approximations.*

Let $\theta_1, \cdots, \theta_n$ be irrational numbers, such that 1, $\theta_1, \cdots, \theta_n$ are linearly independent. It is well known that there are an infinity of simultaneous Diophantine approximations $u_1/u, \cdots, u_n/u$, such that $|\theta_i - u_i/u| < u^{-1-1/n}$, for $i=1, \cdots, n$. The author investigates what precision can be attained by simultaneous approximations, when restrictions are imposed on the signs of the differences $\theta_i - u_i/u$. It is shown that for any $\epsilon > 0$, there are always simultaneous approximations, with *each* difference having a prescribed sign, which satisfy $|\theta_i - u_i/u| < \epsilon u^{-1}$, for $i=1, \cdots, n$. Further there are always an infinity of simultaneous approximations with *one* of the differences having a prescribed sign, which satisfy $|\theta_i - u_i/u| < 2^{1-1/n}u^{-1-1/n}$, for $i=1, \cdots, n$. But if more than one of the n signs are prescribed this extra degree of accuracy is not always attainable. In fact, when two or more of the signs are prescribed, for any continuous function $f(u)$ tending to zero as u tends to infinity, there are irrationals $\theta_1, \cdots, \theta_n$, such that the inequalities $|\theta_i - u_i/u| < u^{-1}f(u)$, $i=1, \cdots, n$, have only a finite number of solutions subject to the prescribed conditions on the signs. The results are obtained mostly as corollaries of a more general investigation of the "asymptotic directions" of n linear forms in $n+1$ integral variables, which will appear in the Proc. London Math. Soc. (Received October 16, 1947.)

92. Ernst Snapper: *The center of the endomorphism ring of a group for which the decomposition theorem holds.*

Let $\mathfrak{V}$ be an abelian group with a commutative ring A as operator domain. Let $\mathfrak{V}$ be a direct sum of cyclic operator subgroups, $\mathfrak{V}=(v_1) \dotplus \cdots \dotplus (v_h)$, where $\mathfrak{a}_i$ is the annihilating ideal of v_i. Suppose that $\mathfrak{a}_1 \subseteq \mathfrak{a}_i$ $(i=1, \cdots, h)$, that is, that $\mathfrak{a}_1$ is the anhilating ideal of $\mathfrak{V}$. It is proved that then the center C of the operator endomorphism ring of $\mathfrak{V}$ consists of the multiplications of elements of $\mathfrak{V}$ with elements of A. Consequently, C is isomorphic with the factor ring $A/\mathfrak{a}_1$. The conditions of this theorem are satisfied, for instance, if A is a principal ideal ring and $\mathfrak{V}$ has a finite number of generators with respect to A. For a finite abelian group $\mathfrak{V}$ without operators, the theorem states that the center C of the endomorphism ring consists of the endomorphisms $v \rightarrow nv$, where n is a fixed whole number and v ranges over $\mathfrak{V}$. Consequently, C is then isomorphic with the factor ring $I/(p)$, where I is the ring of whole numbers and p is the smallest non-negative integer such that $pv=0$ for all $v \in \mathfrak{V}$. (Received October 18, 1947.)

93. Morgan Ward: *Divisibility and factorability properties of general numerical functions.*

Let $W=\phi(n)$ be a function on the positive integers to a fixed residuated distributive lattice $\mathfrak{S}$ with null element O and unit element I in which quotients are unique. ϕ has property D if $\phi(0)=O$, $\phi(1)=I$ and $\phi(n) \supseteq \phi(m)$ whenever n divides m. ϕ has property F if $\phi(nm)=\phi(n)\phi(m)$ whenever n, m are coprime. Among other results it is shown that a necessary and sufficient condition that a function ϕ have both properties D and F is that there exist a second function ψ such that $\psi(nm)=\psi(n) \cup \psi(m)$ for n, m coprime and $\phi(n)=\prod \psi(p^r)$. Here the product refers to multiplication in the lattice, and is extended over the powers of primes dividing n. $\psi(n)$ is uniquely determined by ϕ. (Received October 24, 1947.)

ANALYSIS

94. J. G. Herriot: *Inequalities for the capacity of a lens.*

A lens is a solid determined by the intersection of two spheres. It is the purpose of this paper to compare the electrostatic capacity C of the lens with certain more accessible geometrical quantities. These quantities are the surface radius $\bar{S}$ which is the radius of the sphere having the same surface area as the lens and the "outer radius" $\bar{r}$ of the meridian section of the lens. It is shown that $(4/\pi)\log 2 \leqq C/\bar{r} \leqq 4/\pi$, the extreme values corresponding to two equal tangent spheres and the circular disk respectively. Moreover, $C/\bar{r}$ is greater or less than one according as the dielectric angle of the lens is greater or less than π. It is shown that for two tangent spheres (limiting case of lens) $2^{1/2}\log 2 \leqq C/\bar{S} < 1$, the equality holding only for two equal spheres. Since $2^{1/2}\log 2 = .980$, it is seen that $\bar{S}$ is a close approximation to C in this case. For the symmetric lens $2^{3/2}/\pi \leqq C/\bar{S} \leqq 1$, the extreme values corresponding to the circular disk and sphere respectively. It is conjected that $C \leqq \bar{S}$ for all lenses. (Received October 15, 1947.)

95. M. R. Hestenes: *Quadratic forms in the calculus of variations.*

The second variation $J(\eta)$ of an integral $I(y)$ in the calculus of variations is a quadratic form of the variation η. Moreover these variations with a suitable norm form a Hilbert space, provided the class of variations is taken sufficiently large. In the present paper is studied the properties of quadratic forms in Hilbert space that have significant applications in the calculus of variations together with these applications. This method enables one to give a unified treatment of the second variation for the various simple integral and multiple integral problems that arise in the calculus of variations. (Received October 22, 1947.)

96. Samuel Karlin: *Bases in Banach spaces.*

This paper investigates four types of bases in separable Banach spaces. A basis in a Banach space E is a sequence of elements x_n such that each element x has a unique representation in $x_n(\|x-\sum_1^m a_n x_n\| \to 0)$. An absolute basis x_n is a basis such that whenever $\sum a_n x_n$ converges, then the series converges by every arrangement. It is shown that whenever E possesses an absolute basis and the conjugate space E^* is separable, then E^* is weakly complete. In addition, if E has an absolute basis and E^* is weakly complete, then E^* has an absolute basis. As a consequence, it can be deduced that the space of continuous functions has no absolute basis. A basis is said to be absolutely convergent if $\sum_{m=1}^{\infty}\|f_n(x)x_n\|$ converges for every x in E. A space with an absolutely convergent basis is necessarily finite-dimensional. If M is a proper subset of E possessing a basis x_n and a biorthogonal set f_n then a necessary and sufficient condition that M possess a projection operation on it is that there exist an extension g_n of f_n to E^* such that $\sum g_n(x)x$ converges for every x in E. This result is used to give several criteria that a space E be a conjugate space. Other related results are obtained. (Received October 5, 1947.)

97. Cornelius Lanczos: *Differentiation of a Fourier series.*

The error of a truncated Fourier series of n terms can be estimated in the following form: exp inx times a relatively slowly changing function of x. Differentiation of the first factor brings the large n in front and decreases the convergence of the differentiated series. The operation $[f(x+\pi/n)-f(x-\pi/n)]/(2\pi/n)$, on the other hand, bypasses the first factor (except multiplication by -1) and differentiates the second factor only. As n grows to infinity, the derivative $f'(x)$ is obtained but now without loss in convergence. By this method a function such as x^{-m}, which is not absolutely integrable but the mth derivative of an absolutely integrable function, becomes expanded into a trigonometric series. The method amounts to a multiplication of the standard Fourier coefficients a_k and b_k by the "attenuation factors," $\gamma_k=(\sin \pi k/n)/(\pi k/n)$, or generally the mth power of these factors. This operator has strong smoothing properties in damping out the oscillations of the square wave series and the delta function series and other analogous series. The method is equally applicable to the Fourier integral. (Received October 24, 1947.)

98. E. B. Tolsted: *Limiting values of subharmonic functions.*

Littlewood showed that under certain restrictions a subharmonic function defined in the interior of the unit circle has a radial limit at almost all points of the unit circumference (J. E. Littlewood, *On functions subharmonic in a circle* II, Proc. London Math. Soc. (2) vol. 28 (1928) pp. 383–394.) Priwaloff published a proof that such a subharmonic function has a non-tangential limit at almost all points of the unit circumference (I. Priwaloff, *Sur un problème limité des fonctions sousharmoniques*, Rec. Math. (Mat. Sbornik) Moscow vol. 41 (1934) pp. 3–10). In 1942 Tamarkin discovered a mistake in Priwaloff's theorem, and in 1943 Zygmund constructed a counterexample to the theorem. In this paper counterexamples to the Priwaloff theorem including Zygmund's example are presented; and generalizations of Littlewood's theorem are proved in which the subharmonic function is shown to have a limit for restricted types of approach to the unit circumference. (Received October 1, 1947.)

Applied Mathematics

99. H. W. Becker: *Bridge circuit duality decomposition.*

Let C and C' be any pair of dual bridge circuits. Let C_∞ and C_0 be the fracto components of C with branch b opened and shorted, respectively; and let C_∞' and C_0' be the fracto components of C' with b', dual of b, likewise taken as key branch. Then C_∞ and C_0' are dual circuits, and so are C_0 and C_∞'. More generally, let S and S' be any complete sets of $S-P$ decompositions of C and C', using dual sets B and B' of key branches; then S and S' are dual sets of circuits. (Received October 18, 1947.)

100*t*. S. A. Schaef: *Notes on nonlinear heat conduction.*

Two types of analytic solutions are obtained for the nonlinear heat conduction equation $\partial(K(T)\partial T/\partial x)/\partial x=S(T)\partial T/\partial t$. One of these is a perturbation series, successive terms of which are obtained by the Laplace transform. The other is a set of exact solutions, each containing three arbitrary constants, for various special forms assumed for $K(T)/S(T)$. (Received September 7, 1947.)

Geometry

101. R. M. Winger: *The parametric treatment of cyclic-harmonic curves.*

Moritz, in a series of papers culminating in an extensive monograph (University of Washington Publications in Mathematics (1923)), and Hilton (Ann. of Math. vol. 24 (1922–1923) p. 209) have discussed the curves with the polar equation $\rho = a \cos p\theta/q + k$, where p, q are relatively prime integers and a, k are real. In this paper the author obtains very concise parametric equations of these curves, proving ipso facto that they are rational. He is thus able to apply the technique for dealing with rational curves. The group properties of the curves are considered and several errors of Moritz are corrected. (Received October 20, 1947.)

Logic and Foundations

102. R. M. Robinson: *Recursion and double recursion.*

A simple proof is given of the fact that the function $G_n x$ defined by the double recursion $G_0 x = x+1$, $G_{n+1}0 = G_n 1$, $G_{n+1}(x+1) = G_n G_{n+1} x$ *majorizes* all primitive recursive functions of one variable in the following sense: If Fx is a primitive recursive function, then there is a number n such that $Fx < G_n x$. Also, two primitive recursive functions Ax and $B(x, y)$ are found, such that the function $G_n x$ defined by the double recursion $G_0 x = Ax$, $G_{n+1}0 = 0$, $G_{n+1}(x+1) = G_n B(x, G_{n+1}x)$ *generates* all primitive recursive functions of one variable in the following sense: There exists a primitive recursive function $H(n, x)$, which is indeed a polynomial in n and x, such that if Fx is a primitive recursive function, then there is a number n such that $Fx = G_n H(n, x)$. Results of a similar character were found by Rózsa Péter, *Konstruktion nichtrekursiver Funktionen*, Math. Ann. vol. 111 (1935) pp. 42–60. The proofs given here depend on Theorem 3 in §7 of the author's paper, *Primitive recursive functions*, Bull. Amer. Math. Soc. vol. 53 (1947) pp. 925–942. (Received October 18, 1947.)

103t. Ira Rosenbaum: *On the numbers correlated with p-valued classes of referents and relata with respect to $R_n{}^{p,2(m)}$.*

The class, $sg'R_n'k$, of referents of the individual k with respect to the nth p-valued dyadic relation R_n, is readily determined as the xth of the p^m p-valued classes of an m-membered universe, where $x = 1 + (\sum_{j=0}^{m-1} W_{km-j} \cdot p^{km-j-1})$ and $W_{km-j} = (p^{km-j-1} \vdots (n-1)_p)$. Similarly, the number of the p-valued class, $gs'R_n'k$, of relata of k with respect to R_n is determined as $y = 1 + (\sum_{j=0}^{m-1} W_{mj+k} \cdot p^{mj+k-1})$ with $W_{mj+k} = (p^{mj+k-1} \vdots (n-1)_p)$. Since the number assigned to a sum of classes is determinable by previous methods when the numbers of the summands are known, the number correlated with the class, $R_n''C_z$, of referents of members of C_z is easily obtained, since this class is the logical sum of the classes of referents with respect to R_n of the members of C_z. Similarly determination of the numbers correlated with the classes of relata re R_n of the members of C_z makes possible determination of the number assigned to the class sum of these classes, that is, to the class, $Cnv'R_n''C_z$, of relata re R_n of members of C_z. When $z = 1$, $C_z =$ the universal class, and $R_n''C_z = D'R_n$ (the domain of R_n) while $Cnv'R_n''C_z = \mathrm{Cl}'R_n$ (the converse domain of R_n). (Received October 23, 1947.)

Statistics and Probability

104. G. F. McEwen: *Evaluation of an integral occurring in a fluctuation problem of cosmic-ray showers.*

A. Nordsieck, W. E. Lamb, Jr., and G. E. Uhlenbeck (Physica Vol. 7 (1940)) obtained a solution of the statistical problem of the fluctuation in the number of particles as a function of the thickness, x, of matter penetrated when the effect of ionization is considered. They found for the number of particles, $\bar{N}(z, x) = \exp[-x]$

$\cdot\{1+\int_0 I_1(\eta)\exp[\eta^2/8x]d\eta\}$, where I_1 is the modified Bessel function of the first kind. Various approximation methods, including that of steepest descent, were used to obtain numerical estimates of the definite integral. It can be integrated by repeated applications of a reduction formula involving integration by parts in two ways. Final results are the two converging series: $\exp[-b^2x_1^2]\int_0^{x\leqq x_1} I_1(x)\exp[-b^2x^2]dx=\{f_0(x)+(2b^2x)f_1(x)+(2b^2x)^2f_2(x)+\cdots\}\exp[-b^2(x-x_1)^2]-\exp[-b^2x_1^2]$, and $\exp[-b^2x_1^2]\int_0^{x\geqq x_1} I_1(x)\exp[-b^2x^2]dx=[\exp(x_1/2)-1]([1+f_0(x_1)]/2-\{f_1(x)/(2b^2x)+f_2(x)/(2b^2x)^2+\cdots\}\exp[-b^2(x-x_1)^2]$, where the tabulated function $f_i(x)=\exp(-x)I_1(x)\leqq 1$, and $2b^2x_1=1$, and $[1+f_0(x)]/2=\sum_{i=0}^{\infty} f_i(x)$. (Received October 20, 1947.)

Topology

105*t*. R. F. Arens: *Representation of certain nearly commutative Banach algebras.*

It is proved that a certain type of Banach *-algebra in which $xx^*=x^*x$ is always in the center is equivalent to the ring of all continuous quaternion valued functions (vanishing at infinity) on a locally compact Hausdorff space which satisfy a certain condition of covariance under the ordinary orthogonal group. (Received October 25, 1947.)

J. W. Green
Associate Secretary