the former relation always holds with the equality sign (continuity). A deeper result is the "law of subadditivity": If $C=C'+C''$ and $\delta(C', g)=\delta'$, $\delta(C'', g)=\delta''$, $s(C)=s$ then $(g(\delta)-g(s))^{-1}\leqq(g(\delta')-g(s))^{-1}+(g(\delta'')-g(s))^{-1}$. This leads, in case $q=3$, $g(x)=x^{-1}$, to an improved form of Kellog's rule of conductor capacities. (Received April 26, 1947.)

289. Philip Hartman and Aurel Wintner: *Töplerian* (L^2)*-bases.*

Let $\phi(t)$ be an odd, periodic function of period 2π. The problem of conditions, sufficient or necessary, in order that the sequence $\phi(t)$, $\phi(2t)$, $\phi(3t)$, $\cdots$ form an (L^2)-basis on $(0, \pi)$ is considered. The similar problem of the possibility of an (L^2)-expansion, $f(t)\sim\sum c_n\phi(nt)$, for arbitrary functions $f(t)$ of class (L^2) in $(0, \pi)$ is also treated. The problem is attacked by considering the infinite set of linear homogeneous equations $Dx=0$, where D is the Toeplitz D-matrix associated with the Dirichlet series $\sum\phi_n n^{-s}$. (Received April 15, 1947.)

290. Mark Kac: *Distribution properties of certain gap sequences.*

Erdös and Fortet noticed that for $f(x)=\cos 2\pi x+\cos 4\pi x$ and $n_k=2^k-1$ the distribution function of (*) $m^{-1/2}\sum_1^m f(n_k x)$, $0\leqq x\leqq 1$, cannot approach the normal distribution. In this paper it is shown that the measure of the set of x's for which (*) is less than ω approaches, as $m\rightarrow\infty$, the integral $\int_0^1 k(\omega, x)dx$, where $k(\omega, x) = \pi^{-1/2}\int_{-\infty}^{\omega/|\cos\pi k|}\exp(-v^2)dv$. Actually, a much more general theorem can be established but its statement is too complicated to be included here. (Received April 25, 1947.)

291. Otto Szász: *Quasi-monotone series.*

A sequence $\{a_n\}$ of positive numbers is called quasi-monotone if, for some $\alpha\geqq 0$, $a_{n+1}\leqq a_n(1+\alpha/n)$, $n=1, 2, \cdots$. A series is called quasi-monotone if its terms form a quasi-monotone sequence. Such series have many properties in common with monotone series (a_n decreasing). Thus, (1) if $\sum a_n<\infty$, then $na_n\rightarrow 0$, (2) the series $\sum a_n$ and $\sum 2^n a_{2^n}$ are either both convergent or both divergent. Analogous results hold for infinite integrals. (Received April 11, 1947.)

Applied Mathematics

292. K. O. Friedrichs: *On the non-occurrence of a limiting line in transonic flow.*

In this paper the author treats gas flows around an air foil or in a duct which are subsonic at the beginning and the end and possess supersonic regions adjacent to the boundary. Consider a set of such flows depending on a parameter such as the free stream Mach number M. Then the problem arises whether for a certain value M^* of M a limit line can appear for the first time. It is proved that that cannot occur. To this end the linear hodograph differential equations for x and y as functions of u and v are considered and it is shown that if the Jacobian never vanishes for $M<M^*$, it does not vanish for $M=M^*$, neither at the boundary nor in the interior of the flow. (Received April 11, 1947.)

293. K. O. Friedrichs: *Water waves on a shallow sloping beach.*

The potential function of the motion of water waves on a beach whose slope is an integer fraction of a right angle was first derived by J. J. Stoker and H. Lewy. In the present paper a different representation of this potential is given from which it is possible, with the aid of the saddle point method, to derive an asymptotic representation valid for small slope angles. Simple formulas for the variation of wave length, amplitude and phase with the depth are obtained. The agreement with the results of exact calculations for a beach with a slope of 6° is perfect. (Received April 11, 1947.)

294. K. O. Friedrichs and Hans Lewy: *Dock problem.*

The problem of two-dimensional gravity waves in a half space filled with water, one-half of the surface being covered by a rigid plate called the "dock," requires the determination of an analytic function $\chi(z)$ defined in the lower half space such that $d\chi/dz$ is real on the positive real axis while $d\chi/dz+i\chi$ is negative on the negative real axis and such that χ behaves like ke^{-iz} at infinity when approached in positive real direction while it vanishes at infinity when approached by any other direction. The solution is given explicitly in terms of two analytic functions represented explicitly as complex integrals. One of the functions is bounded at the origin, representing the edge between the dock and the free water surface, the other one having a logarithmic singularity there. The ratio of the amplitudes at the edge and at infinity for the former solution is $1/2^{1/2}$ in agreement with the general result for waves on sloping beaches and with a slope angle $p\pi/2q$ derived in a paper of the second named author. (Received April 11, 1947.)

295. George Pólya: *On virtual masses.*

A solid moves through an ideal incompressible fluid that fills the whole space outside the solid and is at rest at infinity. The motion of the solid is pure translation with velocity 1, the density of the fluid is 1 and its kinetic energy T. The so-called virtual mass of the solid, $2T=M$, depends on the direction of the velocity; the direction is assessed with respect to a coordinate system rigidly tied to the solid. Averaging M over all directions, we obtain $\overline{M}$. There are reasons to conjecture that *of all solids with given volume the sphere has the minimum average virtual mass* $\overline{M}$. This conjecture has been verified for a general ellipsoid and the analogous theorem in two dimensions has been completely proved. (Received April 11, 1947.)

296. J. L. Synge: *Apsidal angles for symmetrical dynamical systems with two degrees of freedom.*

The path of a particle on a surface of revolution corresponds to a geodesic on a certain surface of revolution S for which distance is defined by the Jacobi action. There is direct correspondence between apsides for the two systems, and the apsidal angles are the same. Upper and lower bounds for the apsidal angle are obtained. (Received April 10, 1947.)