ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

Announcement. Beginning with the report of the 1947 Summer Meeting, this BULLETIN will publish the abstracts of papers offered for presentation at a meeting of the Society as part of the report of the meeting. This arrangement will save considerable space in the BULLETIN due to the fact that it will no longer be necessary to print the title of a paper and the name of the author as part of the abstract and also as part of the report of a meeting. The present plan of publishing abstracts was inaugurated in 1930 in the expectation that abstracts would appear in the BULLETIN before the papers were read at meetings of the Society. Unfortunately a large proportion of the abstracts are not received in time to make such advance printing possible.

The Editors are pleased to announce in this connection that the Secretary of the Society plans to have available for distribution at as many meetings of the Society as possible mimeographed copies of the abstracts of papers to be presented. It is believed that such a distribution of the abstracts will be of considerable assistance to easier understanding of the papers presented.

ANALYSIS

288. M. Fekete: *On generalized transfinite diameter.*

The function $g(x)$ is called a generator function if (1) $g(x)$ is continuous for $0<x<\infty$; (2) $g(x)$ is strictly decreasing with x^{-1}; (3) $\lim_{x\to 0}g(x)=\infty$. Let C be an infinite limited-closed point set of a Euclidean space E_q of q dimensions and $g(x)$ a generator function. Put $C_{n,2}g(\delta_n)=$minimum of $\sum_{1\leqq\mu<\nu\leqq n}g([P_\mu P_\nu])$ when $n\geqq 2$ and P_i, $1\leqq i\leqq n$, is a subset of C of $n\geqq 2$ points and $[P_\mu P_\nu]$ denotes distance of P_μ, P_ν. Then $\delta_n=\delta_n(C, g)$, the "diameter of order n of C with respect to $g(x)$" cannot increase as n increases and is positive for $n\geqq 2$. Call $\lim_{n\to\infty}\delta_n=\delta=\delta(C, g)$ the "transfinite diameter of C with respect to $g(x)$." In case $q=2$, $g(x)=\log x^{-1}$, δ coincides with the transfinite diameter introduced by the author (Math. Zeit. vol. 17 (1923)); for $q=3$, $g(x)=x^{-1}$, δ is the generalization of the former notion by Pólya-Szegö (J. Reine Angew. Math. vol. 165 (1931)). For arbitrary $q\geqq 1$ and $g(x)$, $\delta_2(C, g)=s=s(C)=$the span of C. It is obvious that $\delta(C, g)\leqq\delta(D, g)$ whenever C is contained in D (monotonicity) whence $\lim_{\rho\to 0}\delta(C(\rho); g)\geqq\delta(C, g)$ when $C(\rho)$ is the ρ-neighborhood of C. It is easily shown that the former relation always holds with the equality sign (continuity). A deeper result is the "law of subadditivity": If $C=C'+C''$ and $\delta(C', g)=\delta'$, $\delta(C'', g)=\delta''$, $s(C)=s$ then $(g(\delta)-g(s))^{-1}\leqq(g(\delta')-g(s))^{-1}+(g(\delta'')-g(s))^{-1}$. This leads, in case $q=3$, $g(x)=x^{-1}$, to an improved form of Kellog's rule of conductor capacities. (Received April 26, 1947.)

289. Philip Hartman and Aurel Wintner: *Töplerian (L^2)-bases.*

Let $\phi(t)$ be an odd, periodic function of period 2π. The problem of conditions, sufficient or necessary, in order that the sequence $\phi(t)$, $\phi(2t)$, $\phi(3t)$, $\cdots$ form an (L^2)-basis on $(0, \pi)$ is considered. The similar problem of the possibility of an (L^2)-expansion, $f(t)\sim\sum c_n\phi(nt)$, for arbitrary functions $f(t)$ of class (L^2) in $(0, \pi)$ is also treated. The problem is attacked by considering the infinite set of linear homogeneous equations $Dx=0$, where D is the Toeplitz D-matrix associated with the Dirichlet series $\sum\phi_n n^{-s}$. (Received April 15, 1947.)

290. Mark Kac: *Distribution properties of certain gap sequences.*

Erdös and Fortet noticed that for $f(x)=\cos 2\pi x+\cos 4\pi x$ and $n_k=2^k-1$ the distribution function of (*) $m^{-1/2}\sum_1^m f(n_k x)$, $0\leqq x\leqq 1$, cannot approach the normal distribution. In this paper it is shown that the measure of the set of x's for which (*) is less than ω approaches, as $m\rightarrow\infty$, the integral $\int_0^1 k(\omega, x)dx$, where $k(\omega, x)=\pi^{-1/2}\int_{-\infty}^{\omega/|\cos \pi k|}\exp(-v^2)dv$. Actually, a much more general theorem can be established but its statement is too complicated to be included here. (Received April 25, 1947.)

291. Otto Szász: *Quasi-monotone series.*

A sequence $\{a_n\}$ of positive numbers is called quasi-monotone if, for some $\alpha\geqq 0$, $a_{n+1}\leqq a_n(1+\alpha/n)$, $n=1, 2, \cdots$. A series is called quasi-monotone if its terms form a quasi-monotone sequence. Such series have many properties in common with monotone series (a_n decreasing). Thus, (1) if $\sum a_n<\infty$, then $na_n\rightarrow 0$, (2) the series $\sum a_n$ and $\sum 2^n a_{2^n}$ are either both convergent or both divergent. Analogous results hold for infinite integrals. (Received April 11, 1947.)

Applied Mathematics

292. K. O. Friedrichs: *On the non-occurrence of a limiting line in transonic flow.*

In this paper the author treats gas flows around an air foil or in a duct which are subsonic at the beginning and the end and possess supersonic regions adjacent to the boundary. Consider a set of such flows depending on a parameter such as the free stream Mach number M. Then the problem arises whether for a certain value M^* of M a limit line can appear for the first time. It is proved that that cannot occur. To this end the linear hodograph differential equations for x and y as functions of u and v are considered and it is shown that if the Jacobian never vanishes for $M<M^*$, it does not vanish for $M=M^*$, neither at the boundary nor in the interior of the flow. (Received April 11, 1947.)

293. K. O. Friedrichs: *Water waves on a shallow sloping beach.*

The potential function of the motion of water waves on a beach whose slope is an integer fraction of a right angle was first derived by J. J. Stoker and H. Lewy. In the