*Moderna teoria delle funzioni di variabile reale.* By G. Vitali and G. Sansone. (Monografie di matematica applicata per cura del Consiglio Nazionale delle Ricerche.) 2d ed., Bologna, Zanichelli, 1943–1946. Vol. 1, 9+194 pp., vol. 2, 7+511 pp.

The first edition of this excellent treatise was reviewed in vol. 43 (1936) p. 15 and vol. 45 (1939) p. 218 of this BULLETIN. The first volume has not been changed much in the revision; a discussion of the theorems of Egoroff-Severini and Lusin in chapter III seems to be the most important addition. On the other hand, the second volume has grown by more than 200 pages. A completely new chapter on approximation and interpolation has been added. About 80 pages are devoted to the theorem of Weierstrass (the Landau-de la Vallée-Poussin approach for the rational case and that of Dunham Jackson for the trigonometric one), best order of approximation, Chebychev polynomials, the interpolation polynomials of Lagrange and Hermite, Bernstein polynomials, and so on.

There are other important additions. Thus in chapter I a discussion of $L_p$ spaces is added. In chapter II the theory of Fourier series has been enlarged somewhat, the main addition being devoted to $(C, \alpha)$ summability. In chapter III the formula of Mehler has been added as well as a discussion of the zeros of Legendre polynomials. The major addition to this chapter, however, is a discussion of the functions on the sphere as well as $(C, \alpha)$ and Poisson summability of the series of Laplace and of Legendre. Chapter IV on Hermite and Laguerre polynomials is practically unchanged. Finally, in the last chapter on Stieltjes integrals the discussion of characteristic functions (=Fourier-Stieltjes transforms of distribution functions) has been completed by the addition of the Bochner-P. Lévy inversion formula and some convergence theorems. The additions have done much to enhance the value of the treatise which has become a convenient and reliable handbook in classical analysis. The book can be strongly recommended. It is to be hoped for that the misspelling of the name of Walsh will be corrected in the third edition.

EINAR HILLE

*Cours d'analyse mathématique.* By G. Valiron. Paris, Masson, 1942–1945. Vol. 1, 2+522 pp., vol. 2, 2+605 pp.

The two volumes under review constitute a comprehensive account of the following topics: the differential and integral calculus, the general theory of analytic functions of a complex variable, algebraic functions and Abelian integrals, a brief introduction to analytic functions of several variables, the theory of ordinary differential equations in the real and complex domain, a brief introduction to the calculus of variations, differential geometry, partial differential equations of the first and second order. The primary object of these volumes is to give an exposition of the material of the "*certificat de calcul différentiel et intégral.*" However on many occasions the author amplifies the scope of his work and goes out of the immediate confines of his curriculum. In spirit, the treatment is classical and aims at giving a rigorous and at the same time succinct account of the topics treated. The author has attained his goal in a distinguished manner and has made available to the mathematical public a very useful pair of volumes on the subject of classical analysis.

One notable feature of the work is the extensive treatment given to the differential and integral calculus (some three hundred pages, about three-fifths of the first volume). In this way an unequivocally solid foundation is laid for the material that follows. This fact lends significance to the book on the American scene where just such material is basic in the cradle phase of graduate mathematical instruction.

We have remarked that the spirit of the work is classical; there is, however, constant reference to research in mathematical analysis of recent years that keeps these volumes in close contact with modern interest. The general pattern of each chapter is to start with a brief historical account of the topics treated and terminate with reference to recent monographs and special tracts for further study.

The first volume starts with a resumé of the fundamental properties of the real and complex number systems, point sets, sequences, and a sketch of the theory of continued fractions. The second chapter treats systematically infinite series and products. The author then turns to a study of functions of a single variable which includes the usual theorems on continuous and differentiable functions, monotone functions, convex functions, arc length, the Riemann integral and applications, including proofs of the transcendental character of $e$ and $\pi$. There follows a brief treatment of the Lebesgue integral of a complementary nature.

The remaining chapters on functions of a single real variable are devoted to functions defined by series or integrals, Fourier series and orthogonal functions, the reduction and mechanical calculation of integrals.

A careful account is given of functions of several variables and multiple integral theory. These topics are treated in fitting detail. Particular mention should be made of the care used in discussing change of variable in double integrals. Of course these topics are treated under hypotheses which are restrictive enough to allow simple proofs but which are quite adequate for the usual applications.

The author then proceeds to his account of analytic functions of a single complex variable. The major topics treated are: the elementary functions; the Cauchy theory and applications including the calculus of residues; the Weierstrass theory, analytic continuation, and representation theorems for entire functions; conformal mapping including the fundamental mapping theorem, the results of Schwarz concerning polygonal domains, and as an application of the fundamental theorem a study of the elliptic modular function and theorems of the Picard type; elliptic functions; analytic functions defined by integrals and Hadamard's prime number theorem treated by the method of Landau.

The first chapters of Volume II are concerned with the theory of algebraic functions of a single variable and the associated theory of Abelian integrals. Applications are made to plane algebraic curves and the Jacobi inversion problem.

As preparation for the comprehensive treatment of the theory of ordinary differential equations which follows, a study is made of analytic functions of several variables, implicit function theorems, the method of majorants, and in addition a brief account is given of the theory of contact and envelopes.

The theory of ordinary differential equations is treated in both the real and complex domains. The ground covered is extensive and is well suited for a semester graduate course. Topics treated include the elementary theory; classical equations of the first order; the linear theory, and in particular, the problem of isolated singular points in the complex domain (studied by the methods developed by G. D. Birkhoff); nonlinear differential equations in the complex domain; the classical existence theorems; a brief account of the applications of Lie's theory; the theories of Poincaré and Bendixson concerning singularities in the real domain; Sturmian theorems.

The remaining chapters are concerned with the elements of the calculus of variations, the classical differential geometry of curves and surfaces in euclidean 3-space, and partial differential equations of the first and second order.

Even this brief account of the topics treated will indicate the vast labor involved in the preparation of these volumes. Apart from presenting a wealth of material in a concise rigorous manner, the book has the special merit of synthesizing the subject matter of a *Traité d'analyse* with the results of the research of the last decades.

MAURICE HEINS