265. Ira Rosenbaum: *A method of determining the number n correlated with the given truth table of an arbitrary q-ary function in m-valued logic.*

In a previous paper, 1-1 correlation formulae were obtained between the numbers $1, 2, \cdots, m^{m^q}$ and the truth-tables of the m^{m^q} distinct q-ary truth-functional modes of propositional combination of m-valued logic. In the present paper a formula is given for proceeding in the reverse direction, that is, from the given truth-table of an arbitrary q-ary function to the number n correlated with that table. The original simple procedure for going from a given table to the number n associated with it lacked analytical representation. W. V. Quine suggested that this procedure was analogous to transforming the expression for an integer in the m-ary scale of notation into one in the denary scale of notation. This suggestion led to the following result. Let the m^q truth values in the given table be $V_1, V_2, \cdots, V_{m^q}$. From each of these values subtract one to obtain the values $W_1, W_2, \cdots, W_{m^q}$. The latter values are integers x of the range $0 \leqq x \leqq m-1$ rather than, like the V's, of the range $1 \leqq x \leqq m$. Hence the sequence of W's may be regarded as representing in the m-ary scale of notation an integer n. The mode of representation is indicated by the formula $n-1 = \sum_{j=0}^{c-1} W_{c-j} \cdot m^{c-1-j}$ in which $c = m^q$. (Received February 20, 1947.)

Statistics and Probability

266. Z. W. Birnbaum: *Probabilities of sample-means for bounded random variables.*

Lower bounds are given for the probabilities $P(|\overline{X}_n| \geqq t)$, where $t \leqq a$, $\overline{X}^n = (1/n)\sum_{j=1}^{n} X_j$, and $X_1, X_2, \cdots, X_n$ is a sample of a continuous random variable with a probability density $f(X)$ such that: $f(X) = f(-X)$, $f(|X|)$ is a nonincreasing function of $|X|$, and $f(X) = 0$ for $|X| \geqq a$. (Received March 21, 1947.)

Topology

267. Felix Bernstein: *A lattice color problem.* Preliminary report.

The points of the lattice L of all points with coordinates which are integers or are centers of the elementary squares of L are considered as the regions of a color problem. In a partial set S two points A and B are called neighbors if AB does not contain another point of S and if the distance AB is equal either to 1 or to $2^{1/2}/2$ or to $2^{1/2}$. The number of colors required in order that two neighbors may be colored differently is obviously 5 or less. It is shown that 5 colors are necessary. For the proof two methods are used. The one method is based on the studying of color schemes at certain conveniently chosen boundaries in the manner introduced by G. D. Birkhoff. The other method is based on the study of the effect of the "centers" on the coloring of the total neighborhood. The efficiency of each method varies with the nature of the given configuration. (Received March 22, 1947.)

268. R. H. Bing: *A homogeneous indecomposable plane continuum.*

An example is given of a homogeneous bounded nondegenerate continuum which is not a circle. This answers the following question raised by Knaster and Kuratowski in Fund. Math. vol. 1 (1920) p. 223: If a nondegenerate bounded plane continuum is homogeneous, is it necessarily a simple closed curve? This example refutes a statement made in Theorem 5, C. R. Acad. Sci. Paris vol. 219 (1944) p. 543. (Received February 13, 1947.)

269. R. H. Bing: *Extending a metric.*

Suppose that r is a positive integer and $H_1, H_2, \cdots$ are collections of sets such that each pair of points that can be covered by a coherent collection of r or fewer elements of H_{i+1} can be covered by an element of H_i. If P and Q are two points whose sum cannot be covered by any element of H_s but which can be covered by a coherent collection of sets $h_1, h_2, \cdots, h_n$ belonging to $H_{\alpha(1)}, H_{\alpha(2)}, \cdots$, and $H_{\alpha(n)}$ respectively, then $2[1/r^{\alpha(1)}+1/r^{\alpha(2)}+ \cdots +1/r^{\alpha(n)}]>1/r^s$. With the use of this result, it is found that if M is a given metric on a closed subset K of a metrizable space S, then a metric may be assigned to S that will preserve the given metric M on K. (Received March 19, 1947.)

270. Jean M. Boyer and D. W. Hall: *On Peano spaces as continuous images of intervals.*

Let M be a cyclic Peano space and A the set of all points of M which are interior points of arcs of M. Using the definition of M as the continuous image of the unit interval I it is shown that M is the closure of A. Standard arguments, using the same definition, can then be used to prove that A is a closed set. This result yields at once the three point theorem of W. L. Ayres. (Received March 20, 1947.)

271. C. H. Dowker: *Topologies of infinite complexes.*

Infinite complexes have been topologized in different non-equivalent ways by different authors; for example, the natural complex and the geometric complex of S. Lefschetz (*Topics in topology*, Princeton, 1942, p. 9) and the topological polyhedron of J. H. C. Whitehead (Proc. London Math. Soc. vol. 45 (1939) p. 316). It is shown that the three topological spaces, resulting from topologizing an infinite complex in each of these three ways, are of the same homotopy type. It follows that the topological invariance of homology groups, the Hopf mapping theorems, and the Alexandroff Überführungssatz hold equally for all three topologized complexes. (Received March 21, 1947.)

272. Beno Eckmann: *On infinite complexes with automorphisms.* Preliminary report.

Let K be a closure finite complex with an infinite group G of automorphisms without fixed cells, J an Abelian group. An n-cochain f^n in G, that is, a J-valued function of the n-cells c^n of K, is called *G-finite* if, for each c^n, $f(xc^n)=0$ for almost all $x\in G$ (for all except a finite number). With the ordinary co-boundary operation in K these cochains lead to cohomology groups β^n which in general depend upon K *and* G. The G-finite cocycles which are coboundaries of ordinary (infinite) cochains define a subgroup β_0^n of β^n. By methods similar to those used previously by the author (Comment. Math. Helv. vol. 18 (1946) pp. 232–282) the theorem is proved: When K is *acyclic* in dimensions $n<N$ (in the sense of finite chains), then β^n, $n<N$, and β_0^N are determined by the abstract structure of G and J; they may be described in a purely algebraic way by means of J-valued functions of several variables $\in G$. Geometric applications are based on the remark, that in case G has in K a finite fundamental

domain the β^n are the usual cohomology groups of K in the sense of *finite* cochains; a theorem results which for the dimension $n=1$ was already established by Specker (in an as yet unpublished paper). (Received March 21, 1947.)

273. D. W. Hall and D. C. Lewis: *On coloring six-rings.*

Let Q be a hexagon surrounded by a six-ring in a regular map M of n regions. Formulas have been developed (in terms of free chromatic polynomials for regular maps with fewer than n regions) for the number of ways $M-Q$ can be colored in such a manner that any prescribed color scheme appears in the ring. The formulas are valid for any number of assigned colors. These formulas are the basis for a general six-ring reduction formula and also for numerous inequalities involving chromatic polynomials. (Received February 20, 1947.)

274. O. H. Hamilton: *A fixed point theorem for upper semi-continuous transformations on n-cells for which the images of points are non-acyclic.*

It is shown that if T is an upper semi-continuous transformation of an n-cell M into a subset of itself such that for some positive real number d the image $T(P)$ of each point P of M under T is the boundary of a nondegenerate n-dimensional convex continuum containing a simple domain of diameter greater than d, then for some point Q of M, $T(Q)$ contains Q. (Received March 25, 1947.)

275. M. J. Norris: *A note on regular and completely regular topological spaces.*

Given a set of points and two topologies for this set, one topology is said to be stronger than the other if every open set in the latter topology is open in the former. E. Hewitt (Duke Math. J. vol. 10 (1943) pp. 309–333) has shown that the l.u.b. of a completely ordered family of regular (completely regular) topologies is also regular (completely regular). In this paper slight modifications of Hewitt's methods are used to remove the restriction of complete order. (Received March 21, 1947.)

276. M. J. Norris: *On the representation of real numbers by sequences of integers.*

It is well known that there exist one-to-one functions whose domain is the real number system and whose range is contained in the class of all infinite sequences of integers satisfying the following condition. Given a sequence S_0 and a positive ϵ, there exists a natural number N such that $f^{-1}(S)$ differs from $f^{-1}(S_0)$ by less than ϵ when S does not differ from S_0 in the first N positions. Representations by means of decimals or simple continued fractions furnish such functions. This raises the question as to whether or not there exist one-to-one functions with domain and range as above, such that, given a real number x_0 and a natural number N, there exists a positive ϵ such that $f(x)$ does not differ from $f(x_0)$ in the first N positions when x differs from x_0 by less than ϵ. By introducing the usual Cartesian product topology for the class of sequences, the question is easily answered in the negative. (Received March 21, 1947.)

277. H. M. Schaerf: *Generalization of the theory of distance sets to topological groups.* Preliminary report.

Hugo Steinhaus (Fund. Math. vol. 1 (1920) pp. 93–104) has found interesting theorems on sets of distances between the points of two sets A, B of positive Lebesgue measure. In this paper it is shown that Steinhaus' results are special cases of general theorems on sets $B^{-1}A$, where A and B are measurable subsets of positive measure of a topological group with a countable basis. Thereby, the measure function, $m(X)$, may be any one defined on the class of Borel sets, if only $m(gX)$ and $m(Xg)$ are for every group element g absolutely continuous functions of the Borel set X and if every point has a neighborhood with finite measure. (Received March 28, 1947.)

278. H. M. Schaerf: *On A. Weil's condition M.* Preliminary report.

The author proves that the class B of the Borel sets of a topological group G in any neighborhood space with open neighborhoods and with a countable basis satisfies Weil's condition M (that is, the smallest additive class of sets of $G\times G$, containing all combinatorial products $X\times Y$ with X, $Y\in B$, is invariant under the transformations $[(x, y)\rightarrow(y^{-1}x, y)]$ of $G\times G$ onto $G\times G$.) (Received March 20, 1947.)

279. H. M. Schaerf: *On sets of the form $B^{-1}A$ in topological groups.* Preliminary report.

Given a topological group G in a limit space and a measure function $m(X)$ defined on an additive class K of subsets of G and such that $m(Xg)$ is for every $g\in G$ an absolutely continuous function of the measurable set X. The following theorem is proved: Let A be a measurable set such that one of its neighborhoods is contained in a set of finite measure. Let B belong to the smallest additive class K_F of subsets of G, containing both all closed measurable sets and all sets of measure zero. Then the set $B^{-1}A$ contains a neighborhood of every element $g\in G$ such that $m(Ag\cap B)>0$. Therefore, the set $A^{-1}A$ contains a neighborhood of the group unity if $m(A)>0$ and $A\in K_F$. (Received March 28, 1947.)

280. H. M. Schaerf: *On the continuity of measurable mappings of neighborhood spaces.*

Let a measure be defined on an additive class K of subsets of a neighborhood space. Let T be a topological space satisfying the second axiom of countability. Finally, let $f(x)$ be a "measurable" mapping of a set $E\in K$ into T, that is, such a mapping that the image of any open set of T under f^{-1} is measurable. In this paper necessary and sufficient conditions are proved for the validity of each of the following assertions: (a) For every positive number ϵ there is a subset E_ϵ of E such that the measure of $E-E_\epsilon$ is less than ϵ and such that $f(x)$ is continuous on E_ϵ; (b) E_ϵ can be, moreover, taken to be a closed set; (c) $f(x)$ can be, moreover, assumed uniformly continuous on E_ϵ. (Received March 20, 1947.)

281. G. E. Schweigert and G. S. Young: *Remarks concerning invariants for certain finite transformations.* II.

The transformation $T_1(A)=B$ is exactly k to 1; $T_2(A)=B$ is interior and finite. Eleven types of curve, from the arc through rational curves and curves with property N, to curves of any dimension, are considered as to invariance under T_i and T_i^{-1}. Forty-two of these forty-four cases are settled. The negative cases can not be described here. Those with positive solutions admit the following remarks: Under T_2 only the simple closed curve is not invariant (this "backbone" justifies the choice of curves), most of these are invariant under more general transformations; at most

3 curves are invariant under all 4 transformations; many results are from the literature but require considerable adaptation; the rational curves are invariant under T_1 if locally connected. The unrestricted rational curves appear to be a difficult open question for T_1; otherwise (following Whyburn) they are invariant. See our previous abstract (Bull. Amer. Math. Soc. Abstract 52-5-443). (Received March 21, 1947.)

282. M. E. Shanks: *Decomposition homology groups.* Preliminary report.

To each open set O there corresponds a simple decomposition D whose nondegenerate slice is the complement of O. Thus to each open covering there corresponds a decomposition covering. A Čech type theory leads to the usual groups. The homology groups of a closed subset A and the homology groups mod A are obtained as the groups associated with the sublattices of the decomposition lattice "above" and "below" the decomposition whose only nondegenerate slice is A. By lifting the restriction to simple decompositions a large class of new groups is obtained. (Received March 21, 1947.)

283. S. M. Ulam and John von Neumann: *On the group of homeomorphisms of the surface of the sphere.*

It is proved that the group of all homeomorphisms of the sphere preserving the orientation (that is, the component of unity) is simple. More strongly one proves the existence of a number N such that for any two such homeomorphisms S and T, S different from the identity, there exist at most N conjugates of S whose product is T. The method of the proof can be used to establish analogous theorems about groups of homeomorphisms of other manifolds. (Received March 15, 1947.)

284. F. A. Valentine: *The determination of connected linear sections.*

Consider a continuum S in an n-dimensional Euclidean space R_n $(n \geqq 2)$, a continuum being compact and connected. Let x be a point in R_n such that *each* hyperplane through x intersects S in a connected set, and let K denote the set of all such points x. Theorem: *Each component of the set K is convex.* The definition for K differs from that for the Kerneigebiet defined by Brunn (*Über Kerneigebiete*, Math. Ann. vol. 73 (1913) pp. 436–440), in that empty intersections are admissible, and hyperplanes replace the role played by straight lines. Necessary and sufficient conditions are found for a component of K to be closed. Among the theorems proved the following seemed to be of special interest. Designate the convex hull of S by $H(S)$, and let $B(H(S))$ be the boundary of $H(S)$. Definition: A component D of $H(S)-S$ is called a *dent* of S if $D \cdot B(H(S)) \neq 0$. Theorem in R_2, the plane: *Let S be a continuum in R_2 which has m dents. Then if N is the number of components of K the inequality $N \leqq (m^2+m+2)/2$ must hold.* (Received February 11, 1947.)

285. G. T. Whyburn: *On locally simple curves.*

A mapping $f(A)=B$ is locally simple (Morse-Heins) if it is (1-1) on some neighborhood of any point of A. It is shown that a continuum M has a locally simple representation on the circle (l.s.r.c.) if and only if it is the sum of a finite number of doubly extensible simple arcs, that is, arcs ab contained entirely in the interior of some other arc $a'b'$ in M. Every doubly extensible arc ab in a locally connected continuum M is contained in either a (a) simple closed curve, (b) θ-curve, (c) figure 8, (d) dumbbell curve, (e) lariat curve (circle plus radius), or (f) arc joining two end points of M. If

M is without end points, (e) and (f) may be omitted. Also if M is cyclic, ab lies in a cyclic graph in M. Thus a continuum has a l.s.r.c. if and only if it is the sum of a finite number of simple closed curves and dumbbells. A boundary curve B has a l.s.r.c. if and only if it has only a finite number of nodes and no end points. Further, if B satisfies this condition, *any* nonalternating light mapping of the circle onto B is locally simple. (Received March 3, 1947.)

286. G. T. Whyburn: *On n-arc connectedness.*

A simple elementary proof is given for the well known theorem that any two points of a locally connected generalized continuum M which are not separated in M by any set of less than n points can be joined in M by a set of n independent arcs. The proof represents an extension to arbitrary n of the inductive type of reasoning used by the author in simplifying the proof of the cyclic connectedness theorem. It includes an extension to arbitrary locally arcwise connected separable metric spaces S of the theorem of Menger-Nöbeling to the effect that if a compact locally connected space S is n-point strongly connected between two disjoint closed sets P and Q, then S contains a set of n disjoint arcs joining P and Q. (Received March 3, 1947.)

287. R. L. Wilder: *Recognition of manifolds by accessibility properties.*

Schoenflies used an accessibility property to characterize the S^1 in S^2. Generalizing the regular accessibility of Whyburn to a regular r-accessibility, Alexandroff characterized the S^1 in S^n. But regular r-accessibility is not strong enough to characterize the S^k, $k>1$, in S^n. For the latter purpose, if K and E are subsets of a space S, $K \cap E=0$, $p \in K$, then, using compact cycles and homologies, we define p to be (1) *r-accessible* from E if every r-cycle of $E \cup p$ bounds in $E \cup p$; (2) *semi-r-accessible* from E if there exists an f cos $\mathcal{E}$ of S such that every r-cycle of $E \cup p$ in an element of $\mathcal{E}$ bounds in $E \cup p$; (3) *locally r-accessible* from E if $E \cup p$ is r-lc at p. These are topologically invariant positional properties for closed subsets of S^n (or an n-gm). Using these (successively weaker, and all stronger than regular r-accessibility for $r>0$) properties, the general k-gcm can be characterized in the n-dimensional generalized manifold. For example, the only nondegenerate closed proper subsets of S^2 that are both 0- and 1-accessible are the S^0 and S^1; in S^3, if the boundary B of a simply-connected domain is a continuum and 0-, 1- and 2-accessible, then B is an S^2. These results will appear in Chapter 12 of the author's *Topology of manifolds*, to be submitted to the Colloquium Series of the Society. (Received March 20, 1947.)