matrix determined in terms of the field tensor $f_{\mu\nu}$ by $F=\|f^{\mu}_{\nu}\|=\|g^{\mu\sigma}f_{\sigma\nu}\|$ where $g_{\mu\nu}$ are the components of the metric tensor which in the case of special relativity may be reduced to the form $g_{\mu\nu}=0$ for $\mu\neq\nu$, $c^{-2}g_{00}=-g_{11}=-g_{22}=-g_{33}=1$. In case F is a constant matrix, that is, its components are independent of x^{μ} or τ, the solution of (1) may be written as (2) $V=L(\tau)V_0$ where V_0 is a constant one column matrix and $L(\tau)$ is the one-parameter family of Lorentz matrices generated by the infinitesimal Lorentz matrix $1-\epsilon\lambda F$. Unpublished results of O. Veblen, J. W. Givens and the author give a complete classification of the Lorentz transformations in terms of the components of the tensor $f_{\mu\nu}$, as well as a closed expression for $L(\tau)$. The particle orbits may then be obtained by substituting (2) into the definition of V and integrating. In case f^{μ}_{ν} may be written as $f^{\mu}_{0\nu}+f^{\mu}_{1\nu}$ where $f^{\mu}_{0\nu}$ is a constant tensor and $f^{\mu}_{1\nu}$ is slowly varying, then approximations to the solution of (1) may be obtained by transforming (1) into an integral equation and using the classical Picard iteration process. (Received March 25, 1947.)

255. E. A. Trabant: *The Riemannian geometry of the symmetric top.*

The Riemannian geometry of the symmetric top with moment of inertia coefficients A, A, and B, using Euler's angles as coordinates, is developed. The following general theorem for the static space is proven. A necessary and sufficient condition in order that the static Riemannian space be an Einstein space of constant Riemannian curvature with the first covariant derivative of the Riemann symbols of the first kind equal to zero and which can be mapped conformally upon a 3-dimensional flat space is that $A=B$. (Received March 15, 1947.)

Geometry

256. H. S. M. Coxeter: *Continuity in real projective geometry.*

In the presence of the usual axioms of incidence and separation (including one which ensures the compactness of collinear points), the following nonmetrical form of Cantor's axiom of closure suffices to characterize the real projective line: *Every monotonic sequence of points has a limit.* (The words "monotonic" and "limit" are defined in terms of separation alone.) It can be deduced that every point not belonging to a given harmonic net is the limit of a sequence of points of the net, whence the fundamental theorem follows at once. The axioms of Archimedes and Dedekind are likewise deducible. (Received March 22, 1947.)

257. John DeCicco: *Characterization of Halphen's theorem on central and parallel fields of force.*

Halphen showed that if the ∞^5 trajectories of a positional field of force are all plane curves, the lines of force are all straight lines concurrent in a fixed point which may be finite or at infinity, that is, the field of force is central or parallel. The author studies the problem of determining all the positional fields of force whose trajectories are general helices. (A helix is a curve drawn on any cylinder whatever, cutting the generators at a constant angle. In particular, if the curve cuts the generators orthogonally, the curve is plane.) It is proved that if the ∞^5 trajectories of a field of force are all helices, they must be all plane curves, and the field of force is central or parallel. Another characterization was obtained by Kasner. If each trajectory of a positional field of force lies on some sphere or plane, all the trajectories are plane curves, and the field of force is central or parallel. (Received March 6, 1947.)

258. V. G. Grove: *On the Darboux tangents.*

Abramescu (*Sur les tangents de Darboux d'une surface*, Annales Scientifiques Universitatea Jassey, section 1, vol. 27 (1941) pp. 283–288) gave a metric characterization of the cubic curve obtained by equating to zero the terms of the expansion of a surface at an ordinary point up to and including the third order. The author gives a projective characterization of such a cubic curve, and thereby gives a new projective characterization of the tangents of Darboux. By applying the method used in this characterization to the curve of intersection of the tangent plane with the surface, a new characterization of the second edge of Green is found. Finally a simple characterization of the Green-Fubini projective normal in terms of two binary forms is found. (Received March 20, 1947.)

259. C. C. Hsiung: *Differential geometry of a surface at a parabolic point.*

The purpose of this paper is to study the projective differential geometry of a surface in the neighborhood of a parabolic point, at which the two asymptotic tangents of the surface are coincident. The plane section of a surface made by its tangent plane at a parabolic point will be called *the tangential section*. According as the tangential section has a cusp or a tacnode at the parabolic point, we have five essentially different cases. For each case there is obtained a canonical power series expansion of the surface in the neighborhood of the point, together with a geometrical interpretation of the system of reference giving rise to the expansion. In a different manner, I. Popa (Rendiconti del Seminario Matematico della R. Universita di Roma (4) vol. 2 (1938) pp. 136–155) has derived another canonical expansion for one of the five cases, namely, that in which the tangential section has a cusp. (Received March 20, 1947.)

260. Edward Kasner and John DeCicco: *Differential geometric properties of the polar surfaces of a general algebraic surface.*

A general algebraic surface S_n of degree n has $(n-1)$ polar surfaces S_{n-1}, S_{n-2}, $\cdots$, S_1, where the rth polar S_{n-r} is of degree $n-r$, with respect to any point P of space. It is known that if the point P is on the surface S_n, the first polar S_{n-1} and all the other polar surfaces are tangent to S_n at P. The last polar S_1 is the tangent plane. It is shown that the directions of the lines of curvature at the point P of S_n and S_{n-r} are identical, and similarly for the asymptotic directions. The ratio of a principal curvature of S_{n-r} to that of S_n at the point P is $\rho=(n-r-1)/(n-1)$. The ratio of the mean curvatures is the same quantity ρ. The ratio of the gaussian curvatures is $\rho^2=(n-r-1)^2/(n-1)^2$. The ratio of the torsions of the asymptotic lines is also this number ρ. These results are extended to the case of higher order contact. Finally the case has been studied where P is a singular point of the algebraic surface S_n. (See forthcoming paper in Proc. Nat. Acad. Sci. U.S.A. concerning the polar theory of a general algebraic curve of the plane.) (Received March 6, 1947.)

261. B. J. Lockhart: *Covariants of a valence correspondence on an algebraic curve.*

Covariant correspondences of a given valence correspondence T are shown to satisfy certain relations. These relations are used to characterize the nature of T. A cyclic set of n points for T is defined as a set of n distinct points on the curve closed under T. The enumeration of such sets for arbitrary n is carried out for general symmetric and nonsymmetric correspondences. (Received February 28, 1947.)

262. R. M. Robinson: *On the decomposition of spheres.*

According to the "Banach-Tarski paradox" (Fund. Math. vol. 6 (1924) pp. 244–277), it is possible to cut a solid unit sphere into a finite number of pieces, and reassemble these by translation and rotation to form two solid unit spheres. Recently, Sierpinski (Fund. Math. vol. 33 (1945) pp. 228–234) showed that the total number of pieces may be taken as eight, three pieces being used to form one of the new spheres and five the other. In this paper, it is shown that the smallest possible total number of pieces is five; one of these pieces may be taken to consist of a single point. For the surface of the sphere, a similar result is true with four pieces. This follows from the fact that the surface of a sphere can be divided into two pieces, each of which can be subdivided into two pieces congruent to itself. More generally, the surface of a sphere may be decomposed into pieces satisfying any system of congruences, provided that it is not demanded, explicitly or implicitly, that two complimentary portions of the surface be congruent. (Received March 3, 1947.)

Logic and Foundations

263. J. C. C. McKinsey and Alfred Tarski: *Some theorems about the sentential calculi of Lewis and Heyting.*

In this paper the authors prove certain theorems regarding systems of sentential calculus, by making use of results they have established elsewhere regarding closure algebras and Brouwerian algebras. Some of the results are new (in particular it is shown that there are infinitely many functions of one variable in the Heyting calculus, and various theorems about extensions of the Lewis system S4 are obtained); others have been stated without proof in the literature (in particular the authors establish some theorems due to Gödel, which enable one to translate the Heyting calculus into the Lewis system S4). (Received March 21, 1947.)

264. Ira Rosenbaum: *A method of determining the nth q-ary truth-function in m-valued logic.*

The following method is believed useful since truth-functional modes of propositional combination are very numerous in m-valued logic. There are, for example 19,683 distinct binary modes of combination in three-valued logic and m^{m^q} q-ary modes in m-valued logic. Denote these truth-functional modes of propositional combination by F_i, $i=1, 2, \cdots, m^{m^q}$. Let each function be regarded as determined by the truth-values it assigns to each combination of truth-values of its components or arguments. Let $\{n, r\}$ denote the truth-value assigned to the rth of the m^q q-ary truth-combinations by the nth q-ary truth-function, let $\imath x(\cdots)$ denote the one and only object x satisfying the condition $\cdots$, and let $[---]$ denote the integral part of $---$. Then the nth q-ary truth-function of m-valued logic is defined thus: $F_n=(\{n, 1\}, \{n, 2\}, \cdots, \{n, m^q\})$ where $\{n, 1\}=\imath x(1\leqq x\leqq m \cdot\& \cdot x\equiv n \pmod{m})$ and $\{n, k\}=\imath x(1\leqq x\leqq m \cdot\& \cdot x\equiv[(n-\{n, 1\})/m^{k-1}+1] \pmod{m})$, $1<k\leqq m^q$. The above formulae determine the nth q-ary truth-function of m-valued logic without the construction of tables with m^{m^q} columns, provide a simple nomenclature for the numerous functions of m-valued logic, and are valid also for two-valued logic. A simple process for determining n, given the values $\{n, 1\}, \cdots, \{n, m^q\}$, is also available. (Received March 6, 1947.)