If F is called regular if a normal regular matrix exists with field F, a regular field is co-regular; F is regular if and only if $f(x) = \lim x_n$ is continuous on (c) using the "Mazur norm" $\|x\| = \sup |\sum_{k=0}^{n} a_{nk} x_k|$. Subfields of regular fields are regular. Other conditions are given. All Mazur metrics for a given field are equivalent and all fields are separable. A field is a non-reflexive Banach space. A normal conservative matrix sums only convergent sequences if and only if $\|A^{-1}\| < \infty$ where $\|A\| = \sup \sum_{k=0}^{n} |a_{nk}|$. (Received March 20, 1947.)

241. J. E. Wilkins: *Neumann series of Bessel functions.*

Suppose that $|f(t)|$ is integrable over $(0, a)$ and that $t^{-3/2}|f(t)|$ is integrable over (a, ∞) whenever $a > 0$. Let $s = \sum_{n=0}^{\infty} a_{2n+1} J_{2n+1}(x)$, where $a_{2n+1} = \int_{0}^{\infty} t^{-1} f(t) J_{2n+1}(t) dt$, be the Neumann series associated with $f(x)$. It is shown that $s = \{f(x+) + f(x-)\}/2$ at each positive point x in a neighborhood of which $f(t)$ is of bounded variation if and only if $\int_{1}^{\infty} r J_0(xr) dr \int_{a}^{N} f(t) J_0(tr) dt$ converges to zero as a approaches zero and N approaches ∞. (Received March 21, 1947.)

APPLIED MATHEMATICS

242. J. W. Calkin: *Incipient shock waves in one dimension.*

The author considers the following one-dimensional model: a rectilinear cylinder, closed at one end by a piston, and containing a perfect gas. As is in effect known, while a discontinuous increase in piston velocity produces an immediate shock wave, starting at the piston face, a continuous acceleration results in a delayed shock which starts at a positive distance from the piston face, with initial velocity that of sound. The present paper is concerned with the boundary value problem to which the study of the subsequent motion leads, and provides procedures for approximating to the solution. A variant of the hypothesis of a perfect gas is also considered. (Received March 21, 1947.)

243. J. B. Díaz and H. J. Greenberg: *The determination of upper and lower bounds for the deflection of a clamped plate.*

A method is given for the determination of upper and lower bounds for the deflection w at any point of a thin elastic plate of arbitrary shape clamped along its edges and subjected to a distributed load. The method is based on the application of two variational principles: the first is that of minimum potential energy, the second is closely related to Castigliano's principle of minimum complementary energy but does not seem to have been used before in the present form. These principles yield inequalities for an integral of w. By considering two auxiliary loadings of the plate, in addition to the given loading, inequalities are obtained directly for w at any specified point. The bounds are obtained in terms of integrals of certain admissible functions. Explicit iterative formulas are given by means of which sequences of admissible functions can be utilized to successively improve the bounds. (Received March 21, 1947.)

244. J. B. Díaz and Alexander Weinstein: *Schwarz' inequality and the methods of Rayleigh-Ritz and Trefftz.*

It is shown that lower and upper bounds of a quadratic functional can be obtained by a simple and direct application of Schwarz' inequality and Green's formula, the results being equivalent to the application of the methods of Trefftz and Rayleigh-

Ritz. As an example, a lower bound for the Dirichlet integral $D(\phi)$ of a harmonic function ϕ with prescribed boundary values is derived by putting into the Schwarz' inequality $D^2(\phi, \psi) \leq D(\phi)D(\psi)$ an arbitrary harmonic function ψ. This leads, by Green's formula, to the inequality $D(\phi) \geq (\int_C \phi (\partial \psi / \partial n) ds)^2 D^{-1}(\psi)$, which is equivalent to the lower bound found by Trefftz (Proceedings of the Second International Congress for Applied Mechanics, Zürich, 1927, p. 132). (Received March 19, 1947.)

245. W. F. Eberlein: *A note on spheroidal wave functions.*

The equation defining the (axially symmetric) spheroidal wave functions can be written in the form $\{(1-x^2)u'\}' + (\lambda - \alpha^2 x^2)u = 0$, the boundary condition being that $u(x)$ remain finite at $x = \pm 1$. An asymptotic expansion for the characteristic values λ for $\alpha \gg 1$ is obtained. (Received March 21, 1947.)

246. Benjamin Epstein: *On the mathematical description of certain breakage mechanisms.*

It has been observed that the particle size distributions obtained from some breakage processes appear to be logarithmico-normal. In this paper a hypothetical breakage mechanism is constructed which has the property of yielding size distributions after breakage which will more closely approximate a logarithmico-normal distribution the longer the breakage process is continued. More precisely it is proved that if: (a) the probability of breakage of any piece during any step of the process is a constant independent of the size of the piece and the previous breakage history of the piece and (b) the distribution of pieces obtained from the application of a single breakage event to a given piece is independent of the dimension of the piece in the sense that the fraction by weight of material having dimension less than ky $(0 \leq k \leq 1)$ arising from the breakage of a unit mass of size y is independent of y, then the d.f. $F_n(x)$ after n steps in the breakage process is asymptotically logarithmico-normal. The proof depends essentially on the use of the Mellin transform in treating products of random variables. (Received February 18, 1947.)

247. H. J. Greenberg and Witold Hurewicz: *The stability of periodic dynamical processes subject to discontinuous impulses.*

The classical Poincaré-Liapunoff stability theory of periodic motions is extended so as to include the case of dynamical systems subjected to discontinuous impulses: for instance, mechanical systems involving colliding masses. The continuous phases of the motion are assumed to be governed by the equations (1): $\dot{x}_i = f_i(x_1, \cdots, x_m, t)$, $i = 1, \cdots, m$, where the f_i are periodic functions of the time t. In addition, at any t_0 at which the variables x_i satisfy a relation $L(x_1, \cdots, x_m) = 0$ where L is a given function, the x_i are assumed to have prescribed discontinuities $\Delta x_i = \phi_i(x_1, \cdots, x_m)$. With a given periodic motion of such a system a stability matrix is associated. If the characteristic values of this matrix are less than 1 the motion is stable; if one of these is greater than 1 the motion is unstable. Explicit expressions for the elements of this matrix are given in terms of a fundamental system of solutions of the "equations of variation" derived from (1) and of the values of the ϕ_i and their derivatives at the discontinuity points. The general theory is applied to the special case of a system of two degrees of freedom consisting of two linear colliding oscillators. (Received March 21, 1947.)

248. A. E. Heins: *Water waves over a channel with a dock.*

This paper is concerned with the solution of (*)$\Delta\phi(x, y) = 0$ in the strip $-\infty < x < \infty$, $0 \leq y \leq a$ subject to the conditions (a) $\partial\phi/\partial y = 0$ at $y = 0$ for all x, (b) $\partial\phi/\partial y = 0$ at $y = a$ for all $x < 0$, and (c) $\partial\phi/\partial y = \beta\phi$ $(\beta > 0)$ for $y = a$ and all $x > 0$. The velocity potential $\phi(x, y)$ is assumed to possess a logarithmic singularity at $x = 0$, in order that (*) possess a travelling wave term for $x \gg 0$. The surface $y = a$, $x < 0$ is the dock-like surface, while $y = a$, $x > 0$ is the free surface. This problem is formulated as an integral equation of the Wiener-Hopf type in terms of a Green's function kernel which may be exhibited explicitly. The solution under the dock as well as under the free surface is found by a technique which is available for the solution of this integral equation. The equivalent three-dimensional problem is also treated. (Received March 1, 1947.)

249. Rufus Isaacs: *Recent progress in the theory of compressible fluids.*

Until recently progress toward a workable theory of compressible fluids has been barred by the staggering amount of computation required. Modern computing devices, together with theories aimed towards their usage, open up vast possibilities in this and other fields. Chaplygin showed that the differential equations for planar irrotational steady flows become linear when transferred to the hodograph plane. Here the potential and stream functions are functions of the velocity components instead of the point coordinates. Bergman modifies this approach by using not these functions and variables but related ones. For these equations—linear, but still formidable—he has devised an operator which transforms each analytic function of a complex variable into a solution. Analogously in the incompressible case (harmonic solutions) the operator is "Take the imaginary part." The operator involves a sequence of functions which must be known and tabulated in order to solve specific flow problems. The functions are defined recurrently by a relation involving both differentiation and integration. They are being calculated on the Automatic Sequence Controlled Computer at Harvard University in a current research program headed by Professor von Mises. The writer participated last summer and developed the method for this computation. (Received February 28, 1947.)

250. S. C. Kleene: *Analysis of lengthening of modulated repetitive pulses.*

Pulse lengtheners are circuits which lengthen pulses without changing the relative pulse amplitudes. The output of a pulse lengthener as a function of time t is $F(t) = f(\{t\})g(t - \{t\})$ where $f(t) =$ modulating signal (here assumed periodic with fundamental frequency f_m), $g(t) =$ unmodulated lengthened pulse (with repetition frequency f_0), and $\{t\} = [f_0 t]/f_0$. The frequencies which may appear in $F(t)$ are then $|sf_0 \pm pf_m|$ for s, $p = 0$, 1, 2, $\cdots$. This was originally noted for the so-called ideal "box car" lengthener, for which $g(t) \equiv 1$ (in which case the frequencies sf_0, for $s = 1, 2, \cdots$, are absent), by Ming-Chen Wang and G. E. Uhlenbeck (Radiation Laboratory classified report S-10 (May 16, 1944) pp. 103–104). The present paper (based on Naval Research Laboratory classified report R-2555 (July 2, 1945, declassified February 7, 1946)) contains a simple derivation of the Fourier expansion of $F(t)$ for a quite arbitrary $g(t)$, and formulas and graphs for calculating the amplitudes of the terms in the expansion for the family of pulse functions $g(t) = e^{-\alpha t}$ $(0 < t < \beta/f_0)$, $g(t) = 0$ $(\beta/f_0 < t < 1/f_0)$, including the ideal "box car" pulse for $\alpha = 0$, $\beta = 1$. (Received March 5, 1947.)

251. M. Z. Krzywoblocki: *On the two-dimensional steady flow of a compressible viscous fluid far behind a solid symmetrical body.*

Goldstein solved the equations of steady flow of an incompressible viscous fluid in a wake far behind a solid symmetrical body by application of the method of successive approximations. Very far downstream in a wake, if the motion were steady, the assumptions and approximations of the laminar boundary layer theory would be valid. In the present paper this method was applied to the solution of the equations of steady flow of a compressible viscous fluid in a wake far behind a solid symmetrical body. The following equations are taken into account: equations of motion, continuity, energy and state. The coefficients of viscosity and thermal conductivity are assumed to be functions of temperature. Just as in Goldstein's work only the first two approximations are successful, whereas the third one is not successful. (Received March 5, 1947.)

252. Isaac Opatowski: *Application of a theorem of Jacobi to compressible and rotational flows.*

Jacobi has given a method to determine the characteristics of the equation $\sum A^\lambda(x^1, x^2, x^3) \cdot \partial Y/\partial x^\lambda = 0$ if a solution of this equation of the type $Y(x^1, x^2, x^3)$ = const. and a function $M(x^1, x^2, x^3)$ such that $\sum \partial(MA^\lambda/\partial x^\lambda) = 0$ are known (*Vorlesungen über Dynamik*, 2d ed., 1884, pp. 74–79). His method is applied to determine the streamlines of compressible and rotational stationary flows. Then $M = \rho a^{1/2}$, where ρ is the density of the fluid and a is the discriminant of the first fundamental form related to the system of coordinates used (x^1, x^2, x^3). This gives equations of streamlines of rotational flows in the form: $S(x^1, x^2, x^3)$ = const., $W(x^1, x^2, x^3)$ = const., where S is the entropy and W is obtained explicitly by means of M (Jacobi's last multiplier). (Received March 22, 1947.)

253. Edmund Pinney: *Aerodynamically driven oscillations in suspension bridges.*

In the past hundred years many suspension bridges have been destroyed by wind driven oscillations, but most of these failures occurred so long ago and were so poorly observed that their cause remained uncertain. The spectacular and well observed failure of the Tacoma Narrows bridge on November 7, 1940 proved clearly the importance of such oscillations. The present paper develops a method of determining the wind speed at which oscillation sets in. Methods of obtaining symmetrical and asymmetrical vacuum mode forms and frequencies are given. Theodorsen type airforces are used, but a modification is given to include the effects of roadbed slots. A procedure is given to make use of airforce data obtained from wind tunnel section models. The theory is applied to the original Tacoma Narrows bridge and gives a "flutter speed" in close agreement with the observed wind speed at the time of failure. (Received March 20, 1947.)

254. A. H. Taub: *Orbits of charged particles in constant electromagnetic fields.*

The relativistic equations of motion of a particle of charge e and rest mass m_0 in an external electro magnetic field described by the anti-symmetric tensor $f_{\mu\nu}$ (μ, $\nu = 0, 1, 2, 3$) may be written as (1) $dV/d\tau = -\lambda FV$ where V is a matrix of one column $V = \|v^\mu\| = \|dx^\mu/d\tau\|$, $\lambda = e/m_0 c$, c is the velocity of light and F is a four by four

matrix determined in terms of the field tensor $f_{\mu\nu}$ by $F=\left\|f_\nu^\mu\right\|=\left\|g^{\mu\sigma}f_{\sigma\nu}\right\|$ where $g_{\mu\nu}$ are the components of the metric tensor which in the case of special relativity may be reduced to the form $g_{\mu\nu}=0$ for $\mu\neq\nu$, $c^{-2}g_{00}=-g_{11}=-g_{22}=-g_{33}=1$. In case F is a constant matrix, that is, its components are independent of x^μ or τ, the solution of (1) may be written as (2) $V=L(\tau)V_0$ where V_0 is a constant one column matrix and $L(\tau)$ is the one-parameter family of Lorentz matrices generated by the infinitesimal Lorentz matrix $1-\epsilon\lambda F$. Unpublished results of O. Veblen, J. W. Givens and the author give a complete classification of the Lorentz transformations in terms of the components of the tensor $f_{\mu\nu}$, as well as a closed expression for $L(\tau)$. The particle orbits may then be obtained by substituting (2) into the definition of V and integrating. In case f_ν^μ may be written as $f_{0\nu}^\mu+f_{1\nu}^\mu$ where $f_{0\nu}^\mu$ is a constant tensor and $f_{1\nu}^\mu$ is slowly varying, then approximations to the solution of (1) may be obtained by transforming (1) into an integral equation and using the classical Picard iteration process. (Received March 25, 1947.)

255. E. A. Trabant: *The Riemannian geometry of the symmetric top.*

The Riemannian geometry of the symmetric top with moment of inertia coefficients A, A, and B, using Euler's angles as coordinates, is developed. The following general theorem for the static space is proven. A necessary and sufficient condition in order that the static Riemannian space be an Einstein space of constant Riemannian curvature with the first covariant derivative of the Riemann symbols of the first kind equal to zero and which can be mapped conformally upon a 3-dimensional flat space is that $A=B$. (Received March 15, 1947.)

GEOMETRY

256. H. S. M. Coxeter: *Continuity in real projective geometry.*

In the presence of the usual axioms of incidence and separation (including one which ensures the compactness of collinear points), the following nonmetrical form of Cantor's axiom of closure suffices to characterize the real projective line: *Every monotonic sequence of points has a limit.* (The words "monotonic" and "limit" are defined in terms of separation alone.) It can be deduced that every point not belonging to a given harmonic net is the limit of a sequence of points of the net, whence the fundamental theorem follows at once. The axioms of Archimedes and Dedekind are likewise deducible. (Received March 22, 1947.)

257. John DeCicco: *Characterization of Halphen's theorem on central and parallel fields of force.*

Halphen showed that if the ∞^5 trajectories of a positional field of force are all plane curves, the lines of force are all straight lines concurrent in a fixed point which may be finite or at infinity, that is, the field of force is central or parallel. The author studies the problem of determining all the positional fields of force whose trajectories are general helices. (A helix is a curve drawn on any cylinder whatever, cutting the generators at a constant angle. In particular, if the curve cuts the generators orthogonally, the curve is plane.) It is proved that if the ∞^5 trajectories of a field of force are all helices, they must be all plane curves, and the field of force is central or parallel. Another characterization was obtained by Kasner. If each trajectory of a positional field of force lies on some sphere or plane, all the trajectories are plane curves, and the field of force is central or parallel. (Received March 6, 1947.)