in a simple and appealing manner. The reviewer's only regret is that Professor Bliss did not have occasion to include various other topics in the calculus of variations in which he has been interested and to which he has made numerous contributions. The book is a valuable addition to a mathematician's library.

M. R. HESTENES

The theory of potential and spherical harmonics. By W. J. Sternberg and T. L. Smith. (Mathematical Expositions, No. 3.) The University of Toronto Press, 1944. 312 pp. $3.35.

This is a book on a classical field of analysis treated in the classical way and restricted to classical theorems. Potential theory in the 19th century sense is no longer a familiar subject in a mathematical curriculum, and so the value of such a publication is to be considered with respect to a rather special audience. Its needs might equally well be served by a compact presentation in English covering the fundamental notions similar, for instance, to that in Courant-Hilbert, volume 2 (which actually presents a more complete picture as well) or the third volume of Goursat's *Cours d'analyse* with its illuminating problems. The preface suggests the research worker may find some ideas here but this is not borne out by the contents.

The chapter headings indicate the topics taken up: The Newtonian law of gravity, concept of the potential, the integral theorems of potential theory, analytic character of the potential, spherical harmonics, behavior of the potential at points of the mass, relation of potential to theory of functions, the boundary value problems of potential theory, the Poisson integral in the plane, the Poisson integral in space, the Fredholm theory of integral equations, general solution of the boundary value problem. The last chapters deserve commendation, especially for the excellent presentation of the Fredholm theory. The Riemann integral is used exclusively, and only an elementary acquaintance with real variable theory and the rudiments of functions of a complex variable is needed as a prerequisite. The book can be read by the first year graduate student in an American university.

The exposition is notably lucid, and in fact almost conversational in its naturalness, though there is little emphasis. Perhaps the principal results could have been singled out and more said about the power and generality of these theorems. The preface remarks on a "consistent" use of vector analysis. Without implying any criticism, it should be noted, however, that the treatment is not vectorial in spirit. The vectors are usually brought in as shorthand notations for the more complicated Cartesian expressions actually manipulated.

A carping pedant could find details to criticize. We list a few drawn at random. The writers never define *portion* of a region, and presumably intend subregion; they refer to a one-sided surface where one side of a two-sided surface is meant. There is a familiar lacuna on page 175; a line integral is first shown to vanish on all simple closed curves of class C', but this result is later quoted as having been established for *all* simple closed curves. In the reviewer's mind, such defects are of minor importance at this level of maturity.

The criticisms mentioned below are not too serious and can be met by the addition of a few pages of material in a later edition. They may be interpreted in the spirit of the authors' invitation for "suggestions for improving the text," though it will be clear that they derive in part from a somewhat different conception of values in this field. Selection of material is, of course, the author's prerogative and one cannot quarrel with a decision to leave out such topics as tesseral and toroidal harmonics. However, it seems to the reviewer that the book is not complete with an exposition of the classical viewpoint alone, and it is eminently desirable that the reader be given an understanding of what modern work in potential theory deals with and what some of the underlying difficulties are. (The last two chapters constitute a step in this direction, but they require considerably more sophistication than the rest of the book.) In this connection one may perhaps go too far in stressing pathological aspects, but surely a student cannot appreciate potential theory if no pathology is mentioned. Thus, for instance, the simple example of zero Dirichlet data, except at one point where the value is not defined, is omitted, nor is the Zaremba example cited. There is no mention of the Lebesgue spine, or on a higher level, the necessity for analogues of primends. Similarly, the reviewer feels Hadamard's classic example of the failure of the Dirichlet principle should have been given. Nothing is said about the possibility of definitions of assumption of boundary values other than the obvious one, and so the reader remains unaware of the spirit of so many of the contributions of the last 40 years. Here a single result such as that of A. J. Maria's, 1932, Bulletin note, would clarify the subject. The writers do not call attention to the role of convexity which appears already in such early methods as those of Schwarz. In this connection an example of the sort of region for which Poincaré's inequality is invalid would bring home the meaning of the restrictions made throughout the book. Even though Stieltjes or Radon integrals do not appear, perhaps something might have been said about singular types of mass distribution, and some mention made of sub- and superharmonic functions and their implications.

Some small natural additions in connection with applications would have increased the utility of the book considerably for the reader interested in physics. Thus for instance the method of images requires more complete treatment. Practically all the theory is available in the book for consideration of charge distribution on conductors and perhaps even polarization. Considering the details introduced in the treatment of conformal mapping, the Schwarz-Christoffel formula might well have been introduced in view of the applications in hydrodynamics and in the determination of the field in modern physical apparatus such as the cyclotron.

The problems are, in the main, formal exercises. At least a few interesting ones should have been included, say comparable to the reviewer's favorite: the proof of Maxwell's assertion that the inverse *n*th power law of attraction is singled out by the requirement that "similar bodies be similarly charged."

The index could stand amplification. Thus, for example, so important a term as "region" does not occur in the index and indeed is defined just once in the book in a footnote. On the whole, however, the book is meritorious and can be recommended to a student.

D. G. BOURGIN

Elementary matrices and some applications to dynamics and differential equations. By R. A. Frazer, W. J. Duncan, and A. R. Collar. Cambridge University Press, 1946. 14+416 pp. $4.00.

This is a reprint of the book which was first published in 1938 and reviewed in Bull. Amer. Math. Soc. vol. 45 (1939) p. 825. During the past eight years the use of matrices in problems of applied mathematics has become more widespread and part of the credit for this is due the book under review. The theory is well presented but it is regrettable that the authors have felt it necessary, in order to maintain the elementary level mentioned in the title, to refrain from a discussion of the canonical form of a matrix under change of basis and from the proof of the theorem that the number of linearly independent solutions of a system of linear differential equations with constant coefficients is the degree of the determinant of the system. The applications to dynamics (particularly those dealing with flutter problems in aerodynamics) are well chosen and realistic and the book may be recommended to anyone who wishes to know how matrices may be used to advantage in applied mathematics. The printing maintains the high standard set long ago by the Cambridge University Press.

F. D. MURNAGHAN