ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

ALGEBRA AND THEORY OF NUMBERS

103. A. A. Albert: *The Wedderburn principal theorem for Jordan algebras.*

The author has recently given a general theory of Jordan algebras, that is, linear spaces A of linear transformations a, b, and so on, such that A is closed with respect to the operation $a \cdot b = (ab+ba)/2$. In the present article the author proves that the Wedderburn theorem holds for such algebras. This is the theorem stating that if N is the radical of a Jordan algebra A then $A = S+N$, where S is an algebra, and so S is isomorphic to the semisimple Jordan algebra $A - N$. (Received February 11, 1946.)

104. Reinhold Baer: *Absolute retracts in group theory.*

The subgroup R of the group G has been termed a retract of the group G whenever there exists an idempotent endomorphism of G which maps G upon R. As this definition is in strict analogy to the topological concept of retract, one may be tempted to define absolute retracts in like similarity to topological usage. But it shall be shown in the present note that the identity is the only group which is a retract of every containing group. Consequently only modifications of the topological concept will be useful, and it will be shown in this note that each of the following classes of groups may in some sense be termed absolute retract: the complete groups, the abelian groups the orders of whose elements are finite and square free, and the free groups. (Received February 25, 1946.)

105. Grace E. Bates: *Free loops and nets and their generalizations.*

In generalizing to loops the group concept of "freeness," the following definition is used: A loop L is free over its sub-half-loop K (a set of elements in L having the same composition as L, but which is not necessarily closed under this composition), if every homomorphism of K into a loop may be extended to a homomorphism of L into the same loop. The main theorem is the following: If a loop L is free over and generated by its sub-half-loop K, and if S is a sub-loop of L, then S is the free sum of a free loop F and the loop generated by $S \cap K$. Applications of this theorem yield loop analogues to such theorems as Schreier's theorem on subgroups of a free group and the refinement theorem for free products. In the proofs the author uses nets and net constructions, taking advantage of the well known equivalence of net and loop theory. A direct application of one net construction proves the imbeddability of any half-loop into a loop which is free over and generated by the half-loop. (Received March 11, 1946.)

106. H. W. Becker: *Combinatory interpretations of Bell's numbers.*

These are evolved by iterated matrix multiplication of the table of Stirling's numbers, second kind (E. T. Bell, Amer. J. Math. vol. 61 (1939) p. 89; Ann. of Math. vol. 39 (1938) p. 539). They enumerate: (1) the number of distributions of n individuals into crews (or circuits), with m levels of hierarchy (or insulation, or jumpers) in the organization chart; (2) the number of rhyme schemes, or logical relatives (C. S. Pierce, Amer. J. Math. vol. 3 (1880) p. 48) with m plexes of nuance possible to each letter. (Specialized, these yield compositions and partitions of numbers under m modes of addition, as normal, weak, strong, and so on.) The readily formulated breakdowns are formulated according to (1) number of crews, or, equivalently, (2) number of different letters, and to (1) size of first crew, or, equivalently, (2) number of a's. The above may further be interpreted as a generalization of the substitution cycles and inscribed polygons of Touchard (Acta Math. vol. 70 (1939) p. 249): in which the cycles are separated by up to m types of parenthesis; or the polygons are bounded by lines of up to m different colors simultaneously. (Received March 21, 1946.)

107. H. W. Becker: *The general theory of rhyme.*

Prosodic rhyme schemes are called sequations, abstractly, of umbra @. They classify according to range (number of different letters), terminal (last letter), singletons (unrhymed letters, more generally any partition of letters), and quantum (number of a's, more generally any composition of letters). Generalizations are the multipolar, multinomial, multinary, multiplex, and multilinear sequations, having respectively m types of accent, alphabet, partial identity, nuance of nuance, and parallelism. Specialized patrices (expansions of a matrix, with given path restrictions) are those in which: each letter rhymes s times or more, or less; some letter, or none, occurs s-ply; no rhyme repeats within s adjacencies; the differences of successive letters are restricted; given letters are forbidden, or fixed, in certain positions; all numbers in the partition, or range, are odd, or even; there are no nonconsecutive rhymes (these are isomorphic with compositions); the previous is further specialized, to be isomorphic with partitions. In permuted sequations, the debuts of the different letters are in nonalphabetic order: enumerated by differences of zero, integer powers, and the polynomials $(@M+X)^n$. These polynomials kernel the lexicon theorems, by which the patrices are well-ordered. All are generated by variations of $\exp t@ = \exp(e^t-1)$. (Received March 21, 1946.)

108. A. T. Brauer: *On a theorem of M. Bauer.*

In generalization of a theorem of M. Bauer (J. Reine Angew. Math. vol. 131 (1906) pp. 265–267) the following theorem is proved. Let $f(x)$ be a polynomial with integral rational coefficients which has at least one real root. Let $G(k)$ be the group of the residue classes relatively prime to k, and H a subgroup which does not contain the class of numbers congruent to $-1 \pmod{k}$. Then $f(x)$ contains infinitely many prime divisors which do not belong to the classes of H. It follows for instance that each such polynomial contains an infinite number of prime divisors which are quadratic non-residues for a given prime of form $4n+3$. (Received March 20, 1946.)

109. A. T. Brauer and Gertrude Ehrlich: *On the irreducibility of certain polynomials.*

Pólya has proved the following theorem (Jber. Deutschen Math. Verein. vol. 28 (1919) pp. 31–40): If for n integral values of x, the integral polynomial $P(x)$ of degree

n has values which are different from zero and, without regard to sign, less than a certain constant $G_1(n)$, then $P(x)$ is irreducible in the field of rational numbers. Tatuzawa (Proc. Imp. Acad. Tokyo vol. 15 (1939) pp. 253–254) proved that this theorem holds for a larger bound $G_2(n)$. Moreover, he proved that $P(x)$ is also irreducible if $|P(x_\mu)|$ is different from zero and less than a certain constant $H_1(m)$ for m integral values x_μ with $n > m > n/2$. In this paper these results are improved further. Instead of $G_2(n)$ and $H_1(m)$ larger bounds $G(n)$ and $H(m)$ are obtained. These results contain theorems of Schur (Archiv für Mathematik und Physik (3) vol. 13 (1908) p. 367) and of Dorwart and Ore (Ann. of Math. vol. 34 (1933) pp. 81–94). Moreover, an irreducibility criterion of a new type is obtained. If $0 < |P(x_\nu)| < S$ with $S > G(n)$ for n integers x_ν, but less than another smaller constant T for m of these x_ν, then $P(x)$ is irreducible. (Received March 20, 1946.)

110. I. S. Cohen: *Groups and rings of finite rank.* Preliminary report.

An integral domain R is said to be of finite rank if there exists a number k such that every ideal of R has a basis of k elements. It is shown that a necessary condition that R be of finite rank is that every proper residue class ring satisfy the descending chain condition. This condition is also sufficient if R is a local ring, and more generally, if the quotient rings R_P are integrally closed for all but a finite number of prime ideals P; this latter condition is satisfied if the integral closure of R is a finite R-module. These results permit an extension of a theorem of I. Kaplansky concerning R-modules of finite rank. (Received March 23, 1946.)

111. M. A. Coler: *Observations on primes.*

A type of function designated as a *cushion* function is defined. It is shown that certain conjectures, such as Goldbach's Theorem and those concerned with the distribution of twin primes, can be restated conveniently as conjectures regarding the distribution of roots of the same simple cushion function, $\mathcal{C}_1$. (Received February 15, 1946.)

112. H. S. M. Coxeter: *A new extreme form in seven variables.*

Let M be the smallest value taken by a given positive definite quadratic form $F(x_1, \cdots, x_n)$ for integers x_i, not all zero, and let D be the determinant of the coefficients. The form is said to be extreme if the ratio M^n/D decreases for every small change in the coefficients. Extreme forms for $n \leqq 6$ have been completely enumerated. (See Hofreiter, Monatshefte für Mathematik und Physik vol. 40 (1933) pp. 129–152.) Three septenary extreme forms were discovered by Korkine and Zolotareff in 1873. They may be expressed as $\sum_1^7 x_i^2 - x_1x_2 - x_2x_3 - x_3x_4 - x_4x_5 - x_5x_6 - x_jx_7$, where $j = 6$, 5, or 4, and $M^7/D = 2^4$, 2^5, or 2^6. Geometrical considerations now suggest a fourth extreme form $\sum_1^7 x_i^2 + (2/3)\sum_{i<j} x_ix_j - (4/3)(x_5 + x_6)x_7$, for which $M^7/D = 3^7/2^6$. In the corresponding packing of equal spheres, each sphere touches 56 others, as in the case $j = 6$; and yet this packing is denser than in the $j = 5$, where each sphere touches 84 others. (The densest possible lattice packing occurs in the case $j = 4$, where each sphere touches 126 others.) (Received March 6, 1946.)

113. Samuel Eilenberg and Saunders MacLane: *Cohomology theory in abstract groups.* IIa. *Kernels and three-dimensional cohomology.*

In a factor group $E/K = Q$ the elements of Q operate as automorphisms on K, but these automorphisms are defined and multiply only modulo inner automorphisms. A group K together with such operators is called a kernel over Q. For a given abelian group Z with Q as (ordinary) operators, kernels K are considered with center Z. Such a kernel is called extendible if there exists a group E with $E/K=Q$. With the multiplication of kernels suitably defined it is proved that the kernels taken modulo extendible kernels form a group isomorphic with the three-dimensional cohomology group $H^3(Q, Z)$ (Ann. of Math. vol. 46 (1945) pp. 480–509). In particular, the extendible kernels are characterized by the fact that a certain 3-cocycle is a coboundary. (Received March 8, 1946.)

114. Samuel Eilenberg and Saunders MacLane: *Cohomology theory in abstract groups.* IIb. *Group extensions with a non-abelian kernel.*

Let the non-abelian group K with center Z be an extendible kernel over Q, in the sense of the preceding abstract. The totality of all extensions E such that $E/K=Q$ will be enumerated. According to Baer (Math. Zeit. vol. 38 (1934) pp. 375–416) the kernel K determines uniquely a group R with $R/S=Q$ where $S=K/Z$, and every extension E may be regarded as an extension $E'/Z=R$. It is shown that the extensions E' of R by Z so obtained form a coset in the group $H^2(R, Z)$ modulo a well-determined subgroup. Furthermore, different extensions E may lead to identical E'. Each equivalence class of E's obtained in this manner has the structure of a specific group constructed from the one-dimensional cohomology group $H'(Q, Z)$. (Received March 8, 1946.)

115. Samuel Eilenberg and Saunders MacLane: *Cohomology theory in abstract groups.* III. *Non-associative systems and cohomology.*

It is known that the two-dimensional cohomology group $H^2(Q, G)$ (Ann. of Math. vol. 46 (1945) pp. 480–509) can be interpreted as the group of group extensions of the group Q by the abelian group G, realizing the prescribed operators of Q on G. A similar interpretation of the higher dimensional cohomology groups is obtained by the use of loops (for definition, see A. A. Albert, Trans. Amer. Math. Soc. vol. 54 (1943) pp. 507–519). The associator $A=A(a, b, c)$ of three elements in a loop L is defined by $a(bc)=A[(ab)c]$, and the higher associators $A(a_1, \cdots, a_{2n}, b)=A(a_1, \cdots, a_{2n-2}, A(a_{2n-1}, a_{2n}, b))$ are obtained by iteration. A prolongation of Q by G is a loop L with a homomorphism ϕ of L onto Q such that (i) the kernel K of ϕ contains the given group G; (ii) $A(k, b, c)=A(a, k, c)=1$; (iii) each $A(a, b, c)$ lies in the center of the group K; (iv) $ag=[(\phi a)\cdot g]a$. Under a suitable multiplication, the product of two prolongations is again a prolongation. If $A(a_1, \cdots, a_{2n}, b)$ is in G for all b, and is zero for $b\in K$, this associator is essentially a $(2n+1)$-dimensional cocycle f of Q in G. The correspondence $f\longleftrightarrow L$ establishes an isomorphism of $H^{2n+1}(Q, G)$ to a suitably defined group of classes of prolongations. A similar correspondence is established for even dimensions. (Received March 8, 1946.)

116. R. A. Good: *On the construction of clusters.*

In this paper the author continues the study of clusters and discusses their construction by the method of introducing a multiplication operation into an additive group. A suitable multiplicative rule is described if the group is homomorphic to a cyclic subgroup of itself. Clusters whose order is a prime or the product of two primes are characterized. (Received March 20, 1946.)

117. R. A. Good: *On the theory of clusters.*

A cluster is defined as a certain generalization of a (non-associative) ring. A cluster contains its derived ring; the homomorphism concept is applied; ideals are studied, especially the commutator and annihilator ideals. One method of construction is extension of an arbitrary ring by an arbitrary group to form a cluster $\mathfrak{L}$ whose elements are couples. Rules of operation in $\mathfrak{L}$ are described in terms of mappings which are defined in the given systems and subjected to certain interrelating postulates. After the properties and structure of the resulting clusters have been examined, two special cases are treated in detail. Numerous examples illustrate the diversity of properties in various clusters. (Received March 21, 1946.)

118. Lois W. Griffiths: *Linear homogeneous diophantine equations.*

In the system $a_{i1}x_1 + \cdots + a_{in}x_n = 0$ $(i = 1, \cdots, r)$ the coefficients are constant rational integers and the rank of the matrix of the coefficients is r. It is proved that a complete solution of the system in integers is $x_j = (-1)^j t E_j / e$ $(j = 1, \cdots, n)$, in which t is an arbitrary integer and $E_1, \cdots, E_n, e$ are written down directly from the equations in the system in the following way. The equations $\xi_{i1}x_1 + \cdots + \xi_{in}x_n = 0$ $(i = 1, \cdots, n - r - 1)$, in which the ξ_{ij} are arbitrary integers, are adjoined to the original system. Then E_j is the determinant obtained by deleting the jth column of the matrix of the resulting $n-1$ equations, and e is the greatest common divisor of $E_1, \cdots, E_n$. This result is more simple and usable than those in the literature (Th. Skolem, *Diophantische Gleichungen*, Ergebnisse der Mathematik und ihrer Grenzgebiete, vol. 5, no. 4, 1938; D. N. Lehmer, Proc. Nat. Acad. Sci., U.S.A. vol. 4 (1919) pp. 111–114). (Received March 12, 1946.)

119. L. K. Hua: *Geometries of matrices.* III. *Fundamental theorems in the geometries of symmetric matrices.*

The paper contains a proof of the theorem that a continuous mapping carrying symmetric matrices into symmetric matrices and leaving arithmetic distance invariant is either a symplectic transformation or an anti-symplectic transformation. Other results on continuous mappings in the geometry of symmetric matrices are also obtained. The latter half of the paper is concerned with analytic mappings. (Received March 1, 1946.)

120. A. W. Jones: *Semimodular finite groups and the Burnside basis theorem.*

Group theoretic characterizations are given for the following finite groups: those for which the lattice of all subgroups (1) of the entire group is lower semimodular, (2) of the ϕ quotient group is complemented and modular and simple, (3) of the ϕ quotient group is complemented and modular, (4) of the entire group is upper semimodular. As a corollary to one characterization of the groups of (1), it is shown that the direct product of lower semimodular groups, and hence also of modular groups, is always lower semimodular. The groups of classes (2) and (3) are shown to be a proper subclass of those in (1). The groups in (3) are shown to be direct products of groups in (2) having relatively prime orders. By a result of Dilworth it is shown that the groups in (3) constitute the largest class of groups G having G/ϕ lower semimodular to which the Burnside basis theorem applies. A dual Burnside basis theorem is shown to apply to all the groups in (4). (Received March 21, 1946.)

121. G. K. Kalisch: *Uniform spaces and topological groups as generalized metric spaces.*

A *general metric space* has a distance function satisfying the usual conditions, but where the distance is an element of a partially ordered group. It is proved that uniform spaces (cf. A. Weil, *Sur les espaces à structure uniforme*) are general metric spaces whose value group is a certain direct sum of the real numbers indexed according to a directed set. Next it is proved using a theorem of Kakutani (*Ueber de Metrisation der topoligischen Gruppen*, Proc. Imp. Acad. Tokyo vol. 12 (1936) p. 82) that topological groups have a left invariant general metric. This, in turn, implies that topological rings and fields and vector spaces have norms. It is shown by an example that in general the norm of a product need not be less than or equal to the product of the norms (in case the value group is the group of real numbers). It is noted, as a consequence of the first mentioned theorem, that uniform spaces are also sequence spaces where every sequence is indexed according to a fixed directed set (cf. Bourbaki, *Topologie générale: filters*; J. Tukey, *Convergence and uniformity*). (Received March 25, 1946.)

122. Irving Kaplansky: *On a problem of Kurosh and Jacobson.*

In analogy with Burnside's problem, Kurosh (Bull. Acad. Sci. URSS. Sér. Math· vol. 5 (1941) pp. 233–240) has raised the question whether a finitely generated algebraic algebra necessarily has a finite basis. In conjunction with results of Jacobson (Ann. of Math. vol. 46 (1945) pp. 695–707), it is shown that the answer is affirmative if the degrees of the elements are bounded say by n, and if the coefficient field has at least n elements. (Received March 21, 1946.)

123. Fred Kiokemeister: *A note on the Schmidt-Remak theorem.*

A group G with operator domain Ω is said to satisfy the modified maximal condition if every proper Ω-subgroup of G satisfies the maximal condition. A proof of the Schmidt-Remak theorem such as that given by Jacobson (*The theory of rings*, Mathematical Surveys, vol. 2, New York, 1943) yields with few changes the following theorem: Let G be a group with operator domain Ω, and let Ω contain the inner automorphisms of G. Let $G=A_1\times A_2\times \cdots \times A_n$ where the Ω-subgroups A_i are directly indecomposable and satisfy the minimal condition and the modified maximal condition. If $G=B_1\times B_2\times \cdots \times B_n$ is a second decomposition into the direct product of indecomposable factors, then $m=n$. Further the B_i may be so rearranged that $A_i\cong B_i$, and, for $k\leqq n$, $G=B_1\times \cdots \times B_k\times A_{k+1}\times \cdots \times A_n$. (Received March 21, 1946.)

124. Fred Kiokemeister: *A theory of normality for quasigroups.*

A normal relation on a quasigroup G is defined to be an equivalence relation ρ such that (i) $ab\,\rho\,ac$ implies $b\,\rho\,c$, (ii) $ba\,\rho\,ca$ implies $b\,\rho\,c$, (iii) $a\,\rho\,b$ and $c\,\rho\,d$ imply $ab\,\rho\,cd$. The set of elements of G equivalent to an element a under ρ is $R_a(\rho)$. If $R_a(\rho)$ is a quasigroup, it is a normal divisor of G, and ρ is called a-normal. For fixed a, the set of a-normal relations is a modular lattice and is isomorphic with the lattice of normal divisors of G containing a. The Ore theorem on partially ordered sets (Bull. Amer. Math. Soc. vol. 49 (1943) p. 563) is shown to imply the Jordan-Hölder theorem for composition series in G. If e is an idempotent element of G, every normal relation is e-normal. If $A_1, A_2, \cdots, A_n$ are normal divisors of G such that $A_i\cap A_j=e(i\neq j)$ and if $G=A_1\times A_2\times \cdots \times A_n$, then G is said to be the direct product over e of the subquasigroups $A_1, A_2, \cdots, A_n$. The modularity of the lattice of normal relations of G implies the Schmidt-Remak theorem for direct decompositions of G over e through reference to the corresponding structure theoretic theorem by Ore (Ann. of Math. vol. 37 (1936) p. 272). (Received March 21, 1946.)

125. Fred Kiokemeister: *Reducibility in a class of homogeneous polynomials.*

The following theorem is established: Let $H(x_1, x_2, \cdots, x_n)$ be a polynomial, homogeneous of degree n in the variables $x_1, x_2, \cdots, x_n$, with coefficients in the perfect field P. If H factors into a product of linearly independent linear factors over some extension field Δ of P, then H is reducible over an algebraic extension field F of P if and only if there exists in F a nontrivial zero of H. The theorem is proved by exhibiting a commutative linear algebra of which μH is a parastrophic form for some μ in P. The stated property of H follows from a relation between the zeros of H and the ideals of the linear algebra. (Received March 21, 1946.)

126. H. E. Salzer: *Further empirical results on tetrahedral numbers.*

I. The list of integers not greater than n which are the sum of no less than 5 positive tetrahedral numbers was extended from $n=1000$ to $n=2000$. (30 more such integers were found.) The "density" of such numbers decreases at first, but then increases again, showing no tendency to diminish to zero. To illustrate, the number of such integers among the first four groups of 500 are 30, 15, 12 and 18. Only one ends in 4, two in 1, two in 9 and the rest in 2, 3, 7 or 8. II. With the aid of that list, Pollock's conjecture that every integer is the sum of 5 positive tetrahedrals was verified for $n \leqq 20000$ and ending in 2, 3, 7 or 8. With Richmond's statement that every multiple of $5 \leqq 20000$ requires only 4 positive tetrahedrals, this verifies Pollock's conjecture for every $n \leqq 20000$. III. The verification of the author's conjecture that every n^2 is the sum of 4 positive tetrahedrals was extended from $n=200$ to $n=300$. (Received March 13, 1946.)

127. R. D. Schafer: *Equivalence in a class of division algebras of order* 16.

Let $\mathfrak{C}$ be a Cayley-Dickson division algebra over $\mathfrak{F}$. Let $\mathfrak{A}=\mathfrak{C}+v\mathfrak{C}$ with multiplication $(a+vb)(x+vy)=(ax+g\cdot ybS)+v(aS\cdot y+xb)$ where S is the involution $x\leftrightarrow xS=t(x)-x$ of $\mathfrak{C}$ and g is some fixed element of $\mathfrak{C}$. The author has shown (Bull. Amer. Amer. Math. Soc. vol. 51 (1945) pp. 532–534) that $\mathfrak{A}$ is a division algebra of order 16 over $\mathfrak{F}$ for any choice of g in $\mathfrak{C}$ such that $n(g)$ is not the square of an element in $\mathfrak{F}$. Let $\mathfrak{A}_0=\mathfrak{C}_0+v_0\mathfrak{C}_0$ with multiplication defined similarly in terms of a fixed element g_0 of $\mathfrak{C}_0$. The algebra $\mathfrak{A}_0$ is equivalent to $\mathfrak{A}$ if and only if (1) $\mathfrak{C}_0$ is equivalent to $\mathfrak{C}$, the equivalence being given by $x\leftrightarrow xA$, x in $\mathfrak{C}_0$, xA in $\mathfrak{C}$, and (2) $g_0A=\delta^2g$ for some $\delta\neq 0$ in $\mathfrak{F}$. The equivalence between $\mathfrak{A}_0$ and $\mathfrak{A}$ is $x+vy\leftrightarrow xA+\delta v(yA)$. From this it follows that H is an automorphism of $\mathfrak{A}$ if and only if H induces an automorphism A on $\mathfrak{C}$, $(x+vy)H=xA+\delta v(yA)$, and $gA=\delta^2 g$. If $t(g)\neq 0$, then $gA=g$, $\delta=\pm 1$; if $t(g)=0$, then $\delta^4=1$. (Received February 16, 1946.)

128. A. R. Schweitzer: *Sums and products of ordered dyads in the foundations of algebra.* I.

This paper is an algebraic development of a theory of ordered dyads in chapter 2 of the author's *Theory of geometrical relations* (Amer. J. Math. vol. 31). The concept, configuration set of ordered dyads (Bull. Amer. Math. Soc. Abstracts 44-11-474 and 45-9-375) is formally expressed: (1) as an associative and commutative sum of dyads; (2) as an associative and commutative product of dyads. These expressions are respectively generalized to (1) a linear function $\Lambda(\alpha)$ of a complete set of ordered dyads $(\alpha_i\alpha_j)$ on $(n+1)$ elements $(n=1, 2, 3, \cdots)$ with coefficients in a suitable domain D (field F): $\sum\lambda_{ij}\alpha_i\alpha_j$; (2) a "quasi-determinant" $\Delta(\lambda; \alpha)=|\lambda_{ij}\alpha_i\alpha_j|$ whose relation to $\Lambda(\alpha)$ is given by $\Delta(\lambda; \alpha)=\{\Lambda(\alpha)\}^{n+1}/(n+1)!$ where in the product $\{\Lambda(\alpha)\}^{n+1}$ it is assumed that $\alpha_i\alpha_j\times\alpha_k\alpha_l=0$ if and only if $i=k$ or $j=l$. For $n=1$, $\Delta(\lambda;\alpha)=\lambda_{11}\lambda_{22}\alpha_1\alpha_1\cdot\alpha_2\alpha_2+\lambda_{21}\lambda_{12}\alpha_2\alpha_1\cdot\alpha_1\alpha_2$. If the λ's are all equal to unity, then $\Delta(\lambda; \alpha)$ reduces to the symmetric group of configurational sets of dyads (substitutions) expressed as a formal sum. The linear function $\Lambda(\alpha)=\sum\lambda_{ij}\alpha_i\alpha_j$ is used to represent the matrix $\|\lambda_{ij}\|$; representation of the product of two matrices is obtained by multiplying the corresponding linear functions in the usual formal manner and assuming that $\alpha_i\alpha_j\times\alpha_k\alpha_l=0$ or α_{il} according as $j\neq k$ or $j=k$. (Received March 20, 1946.)

129. A. R. Schweitzer: *Sums and products of ordered dyads in the foundations of algebra.* II.

The author discusses properties of quasi-determinants defined in the preceding paper and the relation of the latter to determinants. Quasi-determinants are expanded in two ways: (1) in analogy with the usual expansion of determinants in terms of minors, (2) by multiplying the rows (columns) expressed as formal sums and assuming that $\alpha_i\alpha_j\times\alpha_k\alpha_l=0$ if and only if $i=k$ or $j=l$. The product of two configurational sets of ordered dyads expressed as quasi-determinants is defined to be the quasi-determinant of the product of the two sets expressed as sums corresponding to matrices. This product is generalized to the product of two quasi-determinants: $\Delta(\lambda; \alpha)\cdot\Delta(\mu; \alpha)$ by multiplying the corresponding expansions in the usual formal manner and replacing formal products of configurational sets by their values. Necessary and sufficient conditions are investigated that the product of two quasi-determinants of given order be again a quasi-determinant of the same order. Elementary properties of quasi-determinants are pointed out, such as formal invariance under interchange of rows and columns, commutativity of any two rows (columns) and so on. (Received March 20, 1946.)

130. I. E. Segal: *Representation of certain commutative Banach algebras.*

A theorem of Gelfand and Neumark (Rec. Math. (Mat. Sbornik) N.S. vol. 12 (1943) pp. 197–213, Lemma 1) is given a full proof and extended to the case of an algebra which does not necessarily contain an identity, this case being of interest in the theory of group algebras. The extended theorem is as follows: Let A be a complex commutative Banach algebra on which there is defined a semi-linear operation $a\rightarrow a^*$ which is involutory and bounded, and such that $\|aa^*\|=\|a\|\cdot\|a^*\|$. Then there is an algebraic isomorphism between A and the algebra of all continuous complex-valued functions on a locally compact Hausdorff space Γ, which are continuous and vanish at infinity (in the sense that the subset of Γ on which the absolute value of the function is not less than any preassigned positive number is compact); and if one denotes the function corresponding to a by $a(\gamma)$, $a^*(\gamma)=\overline{a(\gamma)}$ and $\|a\|=\sup_{\gamma\in\Gamma}|a(\gamma)$. The proof includes a simple derivation of the extension of the Weierstrass approximation theorem to functions on compact spaces (due to M. H. Stone), this extension being based on the Riesz-Markoff representation theorem for linear functionals on a space of continuous functions. (Received March 22, 1946.)

131. I. E. Segal: *The spectrum of a commutative subalgebra of a group algebra.* Preliminary report.

The following theorem is proved and applied to the theory of group algebras: Let A be a commutative self-adjoint algebra of bounded operators on a Hilbert space, and suppose that A is also a Banach algebra with respect to a norm N. Assume further that $\|a\| \leqq N(a)$ for every $a \in A$, where $\|a\|$ designates the operator bound of a, and that $aa^*(i+aa^*)^{-1} \in A$, where i is the identity operator. Then every maximal ideal in A is the intersection of A with a maximal ideal in the uniform closure of A. Moreover, $\lim_{n\to\infty}[N\{(aa^*)^n\}]^{1/n} = \|a\|$. The proof is based partly on the characterization of the algebra of continuous functions stated in the above abstract, and partly on the fact that A is adjoint semi-simple (that is, the intersection of all ideals in A which are maximal with respect to being self-adjoint is the null ideal). As applications, several theorems about harmonic analysis on groups which are either locally compact abelian or compact are derived, notably a representation theorem of positive definite functions which in the locally compact abelian case is equivalent to the Herglotz-Bochner-Weil theorem. (Received March 22, 1946.)

132. I. M. Sheffer: *Note on multiply-infinite series.*

It is natural to inquire what properties of simple series extend to multiply-infinite series. In each case the answer depends on the definition of convergence adopted. The present note considers Pringsheim convergence and σ-convergence (cf. Amer. Math. Monthly vol. 52 (1945) pp. 365–376), relative to these two properties: (a) the Riemann rearrangement theorem; (b) the Cauchy product theorem. It is shown that for the former definition neither of these properties is true, whereas both are valid for σ-convergence. (Received March 21, 1946.)

133. J. D. Swift: *Certain power series with coefficients in a finite field.* Preliminary report.

Series of the type $\sum a^{k(k-1)/2}x^k$ where a is a nonzero mark of a finite field are generated from the usual series for e^x by ordering, to the integer k, the mark a^{k-1} in a manner analogous to that used by Morgan Ward in *A calculus of sequences* (Amer. J. Math. vol. 18 (1936) pp. 255–266). These series display properties analogous to those of the exponential series and can also be related to the power series forms of the theta-functions. Since these series may be formally expressed as the ratio of two polynomials, the analogous theta functions and their derivatives may also be expressed in this form. The polynomial appearing in the numerator, $\sum_0^{q-1} a^{k(k-1)/2}x^k$, where q is the number of nonzero marks of the field, has no zeros in the field if a is a primitive mark. Formulas are also obtained for the value of the polynomial when x is replaced by a mark of the field. Polynomials which factor completely in a field are obtained, exhibiting the zeros of the corresponding theta functions. (Received March 19, 1946.)

134. Bernard Vinograde: *Sfield (division ring) composites.*

A decomposition into a ring R and set N, invariant under automorphisms and non-intersecting, of a ring which is sfield modulo its radical (or an ideal) corresponds to the notion of the ring as a configuration in a space where a point or spectrum is a sheaf $\cdots A, B, C, \cdots$ of formally distinct isomorphic sfields subject to $a_ib_j - c_ic_j$

being nilpotent (or in the ideal), where the subscripts tag corresponding images a in A, and so on. Two spectra S and Σ may be subjected to a generalized calculus guided by a distinct master sfield M. A cleft R is generated by one spectrum. If R properly contains a one-sfield spectrum it must be uncleft. The same holds if two spectra (relatively unordered) have overlapping images in M. The singular case of one element generation is easily handled in the case of algebras. The spectra fall into invariant subspaces, and an M without automorphisms is equivalent to one-spectrum subspaces. A spectrum may sometimes be generated by a function $k(x, t)$, where the parameters x generate a sfield and the t generate the equivalence classes. (Received March 22, 1946.)

135. Morgan Ward: *Note on the order of the free distributive lattice.*

If r_n denotes the order of the free distributive lattice on n elements, and if we set $\log_2 r_n$ equal to $2^n\phi(n)$, then for large n, $1/n^{1/2}<\phi(n)<1/4$ so that $\log_2 \log_2 r_n \sim n$. Computational evidence and combinatorial arguments suggest that $n^{1/2}\phi(n)\rightarrow\infty$, but the exact order of $\phi(n)$ is unknown. Incidentally the value of r_6 was computed. It is 7,828352. The method of computation devised easily verified Randolph Church's value 7579 for r_5 (Duke Math. J. vol. 6 (1940) pp. 732–734) but is not powerful enough to evaluate r_7 without prohibitive labor. (Received March 22, 1946.)

Analysis

136. R. H. Bing: *Converse linearity conditions.*

An example is given of a bounded function $f(x)$ $(a<x<b)$ having a derivative on its range and being nonlinear on every subinterval of its range which is such that each point of the graph of $f(x)$ and each point between two points of the graph of $f(x)$ is halfway between some two points of the graph of $f(x)$. (Received March 16, 1946.)

137. Garrett Birkhoff and L. J. Burton: *A weakening of the Hölder conditions for Newtonian force fields.*

Let $\rho(x)$ be a continuous density function of position $x=(x_1, \cdots, x_n)$ near a point $a=(a_1, \cdots, a_n)$ in Euclidean n-space. It is shown that the improper integrals $\int\int\int\rho(x_i-a_i)dR/r^n$ defining the force components for Newtonian attraction exist as improper Riemann multiple integrals (that is, are absolutely integrable) if and only if $\int\int\int\rho d\omega dr/r^{n-1}<+\infty$, where $d\omega$ denotes infinitesimal spherical area. The sufficiency of the usual Hölder conditions for convergence is a weak corollary of this. If $\rho dr d\omega=dm$, the corresponding result for Stieltjes integrals is obtained. (Received March 25, 1946.)

138. D. G. Bourgin: *Approximate isometries.*

The Hilbert space results of Hyers and Ulam (Bull. Amer. Math. Soc. vol. 51 (1945) pp. 288–292) are extended to the spaces $L_p(0, 1)$, $1<p<\infty$. (Received March 22, 1946.)

139. R. C. Buck: *An extension of Carlson's theorem.*

Let $K^*(a, c)$ be the class of functions regular and of order 1 in $R\{z\}\geqq 0$, and of type a on the whole positive real axis and type c on the imaginary axis. If A is a subset of the set I of all integers, denote by $\gamma(A)$ the least number for which the following theorem is true: if $f(z)\in K^*(a, c)$, $c<\gamma(A)$, and if $f(z)$ vanishes in A then it vanishes