and its solutions are elements $\mu \in M$, where $\phi(\mu)=0$ for all $\phi \in S$. An "auxiliary" ideal $\mathfrak{a} \subseteq P$ corresponds to S. The general point ξ of an $(n-i)$-dimensional associated prime $\mathfrak{p}$ of a primary component $\mathfrak{q}$ of $\mathfrak{a}$ determines a "general" $(n-i)$-dimensional exponential solution $\exp(\xi) \in M$ of S. The differential exponent of $\mathfrak{p}$ is δ if a polynomial f with i variables and with coefficients in the field of rational functions of $\mathfrak{p}$, where $f \exp(\xi)$ is a solution of S, has at most degree $\delta-1$. The relationships are studied between δ, the ordinary exponent ρ of $\mathfrak{q}$, the Hentzelt exponent ν of $\mathfrak{q}$, and the multiplicity σ of the root which corresponds to $\mathfrak{p}$ of the $(n-i)$-dimensional elementary divisor of $\mathfrak{a}$. When $\mathfrak{p}$ is isolated and either $p=0$ or $\delta<p$, $\rho=\nu=\sigma=\delta$. When $p\neq 0$, and $\delta \geqq p$, $\rho \leqq \nu$, $p \leqq \nu$, $p \leqq \sigma$. (Received January 17, 1945.)

59. W. J. Sternberg: *On a special set of algebraic nonlinear equations.*

Some physical problems lead to algebraic nonlinear equations, such as the following set $\sum_{i=1}^{3} 1/(x_i/s_k+jy_i)=1/(a_k/s_k+jb_k)$, where $k=1, 2, 3$, $j=(-1)^{1/2}$, $s_1^2\neq s_2^2$, $s_2^2\neq s_3^2$, $s_3^2\neq s_1^2$. An analogous set of two or four or more equations could also be treated. The unknowns are x_i, y_i, while a_k, b_k, s_k are given. The above complex equations are equivalent to six real equations. The transformation $u_i=y_i/x_i$, using the abbreviation $1/(1+s_k^2u_i^2)=f_k(u_i)$ $[i, k=1, 2, 3]$, leads to $\sum_{i=1}^{3} f_k(u_i)/x_i=A_k$, $\sum_{i=1}^{3} u_i f_k(u_i)/x_i=B_k$ where A_k, B_k can be computed from a_k, b_k, s_k. Since the above equations are linear with respect to $1/x_i$, eliminate these unknowns and obtain for the u_i three nonlinear equations, whose left sides are determinants. These determinants are rational alternating functions of the u_i. Simplify the said equations and introduce the elementary symmetric functions w_1, w_2, w_3 of the u_i. The point is that finally three linear equations are obtained for w_1, w_2, w_3. They are uniquely determined and the equation with the roots u_1, u_2, u_3 can be found. The problem is therefore reduced to one equation of third degree. (Received January 20, 1945.)

60. H. S. Wall: *Polynomials with real coefficients whose zeros have negative real parts.*

Let $P(z)=z^n+a_1z^{n-1}+a_2z^{n-2}+\cdots+a_n$ be a polynomial with real coefficients, and let $Q(z)$ be the polynomial obtained from $P(z)$ by dropping out the first, third, fifth, $\cdots$ terms of $P(z)$. Then, the zeros of $P(z)$ all have negative real parts if and only if the successive quotients obtained in applying to $P(z)$ and $Q(z)$ the euclidean algorithm for finding the greatest common divisor of two polynomials have the form c_1z+1, c_2z, c_3z, $\cdots$, c_nz, where c_1, c_2, $\cdots$, c_n are positive. On the basis of this result, a new proof is obtained of a theorem of A. Hurwitz (*Werke*, vol. II, p. 533 ff.). (Received January 24, 1945.)

ANALYSIS

61. Felix Bernstein: *The integral equations of the theta function.*

In 1920 the author showed that the integral equation $\vartheta*\vartheta-2t\vartheta+\vartheta*1-1=0$ $(\vartheta*\vartheta=\int_0^t\vartheta(\tau)\vartheta(t-\tau)dt)$ for the variable $h=e^{-n^2t}$ has the theta zero function $\sum_{-\infty}^{+\infty}h^{n^2}$ as the only solution analytical and regular in the unit circle at the origin. In subsequent papers it has been brought out that this equation defines relationships of the theta zero function to the theory of heat and a number of new theorems. The transcendent theorems of addition have been derived. It has been shown that the Volterra theory of integral equations of this type can be essentially generalized to include functions as solutions whose domain of existence is limited. No relation could be established between the above equation and the central analytical property of the functions, namely the product representation of Jacobi. A new integral equation has been discovered of a different type which exhibits the product representation of one of the four fundamental theta functions by aid of the "Faltung" of two others. These new integral equations are related to Riemann's zeta function. (Received January 23, 1945.)

62. H. L. Garabedian: *Stieltjes integral transformations.* Preliminary report.

This paper involves a study of the integral transformation $v(x)=\int_0^\infty u(y)d\phi(y/x)$, which, with appropriate restrictions on the mass function $\phi(x)$, defines a regular method of summation $(H, \phi(x))$. Various formulations of inclusion relationships between H-methods of summation together with numerous examples are given. These formulations involve a Stieltjes integral equation of the type $\phi_a(u)=\int_0^\infty \phi_a(u/v)d\phi_b(v)$, sequences of the type $\int_0^\infty u^n d\phi(u)$ $(n=0, 1, 2, \cdots)$, and moment functions of the type $\int_0^\infty u^z d\phi(u)$, $\mathfrak{R}(z)\geqq 0$. (Received January 2, 1945.)

63. Dunham Jackson: *The boundedness of certain sets of orthonormal polynomials in one, two, and three variables.*

A recent paper (Duke Math. J. vol. 11 (1944) pp. 351–365) has discussed the boundedness of orthonormal polynomials for which the domain of integration is (among other possibilities) an arc of a plane curve of the second degree. The corresponding problem is studied now for a curve of the form $y=x^m$ in which m is an arbitrary positive integer or has one or another of a few illustrative fractional values. When the exponent is a fraction there is an intimate connection with the theory of orthogonal polynomials in a single variable with auxiliary conditions (Trans. Amer. Math. Soc. vol. 48 (1940) pp. 72–81), calling for new developments of the latter theory. The length of the analysis which appears to be required in the case of even a very simple fractional exponent suggests that the problem as formulated for algebraic curves of any degree of generality is far from elementary. The methods used in connection with plane curves are found to be effective also for skew curves of corresponding simplicity in space of three dimensions. (Received December 11, 1944.)

64. E. R. Lorch: *The Cauchy-Schwarz inequality and self-adjoint spaces.*

The inequality of Cauchy-Schwarz is expanded to the following form. Let s_{ij} denote a skew-symmetric complex matrix and let $\phi(x)$ denote a real function of the complex vector $x=(x_1, \cdots, x_n)$. Suppose that $\max_y \left| \sum x_i\bar{y}_i + \sum s_{ij}x_i\bar{y}_j \right| / \phi(y) = \sum x_i\bar{x}_i/\phi(x)$. Then $s_{ij}=0$ and $\phi(x)=k(\sum x_i\bar{x}_i)^{1/2}$ where k is a positive constant. This result is pivotal in the proof of the fact that if for a linear vector space $\mathfrak{B}$ the equality $\mathfrak{B}=\mathfrak{B}^*$ holds, where $\mathfrak{B}^*$ is the space adjoint to $\mathfrak{B}$, then $\mathfrak{B}$ is either a unitary space or a Hilbert space. The equation $\mathfrak{B}=\mathfrak{B}^*$ is defined to have the following significance: There exists an algebraic and topological isomorphism T between $\mathfrak{B}$ and $\mathfrak{B}^*$, $Tf=f^*$, $f\in\mathfrak{B}$, $f^*\in\mathfrak{B}^*$ such that $(f, f^*)=\|f\|\cdot\|f^*\|$. Here (f, f^*) denotes the value of the functional f^* on the vector f. (Received January 2, 1945.)

65. Harry Pollard: *A theory of the Laplace integral.* Preliminary report.

It is easily verified that under simple conditions on $f(x)$ the formula $f(x) = \int_0^{k/t}(1-xt/k)^{k-1}L_{k,t}(f)dt$ is an identity, where $L_{k,t}(f) = (-1)^k k^{k+1}[k' t^{k+1}]^{-1} \cdot f^{(k)}(k/t)$ is the Post-Widder operator. If $k \to \infty$, $f(x) = \int_0^\infty e^{-xt}g(t)dt$, where $g(t) = \lim_{k\to\infty} L_{k,t}(f)$. Obtaining conditions for which this passage to the limit is valid is tantamount to establishing a representation theory for the Laplace integral. The final results coincide with Widder's theory, but new light is shed on the central role played by the operator $L_{k,t}(f)$. (Received January 20, 1945.)

66. P. R. Rider: *A new use for tables of the incomplete beta function.*

The solution of the problem of minimizing a definite integral having an integrand of the form $(1+y'^2)^m/y''$ has been given. This paper points out how tables of the incomplete beta function can be used to advantage in carrying out the solution in a numerical case. (Received January 22, 1945.)

Applied Mathematics

67. C. A. Truesdell: *The membrane theory of shells of revolution.*

The differential equations of the bending theory, and hence of the membrane theory of shells of revolution, are derived as consequences of the equations of three-dimensional elasticity and certain additional assumptions. Less restrictive than customary boundary conditions at the apex of closed domes are proposed. By means of stress functions satisfying a simple ordinary differential equation, solutions of the stress equations are surveyed, and it is shown that the new boundary conditions can be satisfied for a large class of surfaces for which the old could not. Displacement functions are introduced which reduce the integration of the displacement equations to the integration of a fairly simple ordinary differential equation. Numerous exact solutions of the differential equations are given explicitly, with the aid of the stress and displacement functions, and some numerical examples are given. (Received January 10, 1945.)

Geometry

68. S. B. Jackson: *The four-vertex theorem for surfaces of constant curvature.*

In an earlier paper by the writer (S. B. Jackson, *The four-vertex theorem for spherical curves*, Amer. J. Math. vol. 62 (1940) pp. 795–812) it was shown that on any simple closed spherical curve of class C''', not a circle, there are at least four geodesic vertices, that is, extrema of the geodesic curvature. The aim of the present paper is to extend this result to any surface of constant curvature. Specifically, it is shown that every simple closed curve of class C'', not a geodesic circle, in a simply connected region of a surface of constant curvature has at least four geodesic vertices. The technique of the paper is to map the given region of the surface onto the plane in such a way that the geodesic vertices of the given curve go into the vertices of the corresponding plane curve. The theorem then follows from the known facts about the vertices of plane curves. It is shown by example that the restriction that the curve be in a simply connected region of the surface is essential. Finally, it is proved that the theorem