phenomena of the weight field of force of the earth, and is, in general, a summary of the author's earlier investigations in this field.

The earth's weight field of force is defined by the acceleration relative to the solid part of the earth which a body resting near the surface of the earth sustains, due to gravitation of the earth and the heavenly bodies, and due to the earth's motion in the absolute system.

The first three introductory sections of the monograph deal with the laws of relative motion and the effects of a moving atmosphere. In the fourth section it is shown that the tidal effects and the results of precession and nutation are negligible as compared with the gravitational attraction of the earth and the centrifugal force due to the earth's rotation. This main part of the weight field is at rest with respect to the solid part of the earth, and it possesses a potential. In the next two sections the moments actuating the Eötvös torsion balances are derived in geometrical and analytical treatment. The seventh section and the appendix are devoted to some dynamical phenomena of the weight field which lead to differential equations that can be easily integrated.

In view of the recent revival, due to the national defense program, of interest in geophysics and ballistics, this monograph may prove useful in putting some special investigations, which are capable of application in these fields, in a more available form.

MICHAEL GOLOMB

Introduction to Abstract Algebra. By C. C. MacDuffee. New York, Wiley, 1940. 7+303 pp. $4.00.

The rapid advances in algebra within the last few years have been largely due to an exploitation of the powerful methods of abstract algebra. Accordingly there has been a tremendous increase in the interest in this subject, so that most colleges and universities offer at least one course in abstract algebra. However, students with only an undergraduate course in the theory of equations as a background in algebra frequently have considerable difficulty with the available texts—not so much in reading the proofs as in grasping the significance of the abstract theories. The present book was written primarily as a text for beginning graduate students and is designed to fill the gap between the usual text in the theory of equations and the more advanced texts on algebra. It should prove to be a valuable addition to the growing list of texts on abstract algebra.

The material is organized in such a way that concrete instances of a theory are always at hand before the abstract formulation is made. How this is done, as well as the general scope of the book, will be indicated by brief summaries of the different chapters.

Chapter I. The Theory of Numbers. The book begins with a statement of Peano's postulates for the positive integers, followed by a rapid derivation of the simplest properties of the positive integers and an introduction of zero and the negative integers in terms of pairs of positive integers. Incidentally, rational numbers are not presented from this abstract point of view until Chapter V, although it is assumed in Chapters III and IV that the student has an intuitive knowledge of the rationals. The remainder of the chapter is devoted to a study of such topics as the fundamental theorem of arithmetic, congruences in one or more unknowns, residue classes, theorems of Fermat and Euler, primitive roots, quadratic residues and the law of quadratic reciprocity.

Chapter II. Finite Groups. After the definition of a *group* and derivations of a few properties of groups in general, the rest of the chapter deals only with finite groups. The set of automorphisms of a mathematical system is used as an important illustration of a group. A body of theorems is established, culminating in the Jordan-Hölder Theorem, which is essential for the Galois theory of the succeeding chapter. Direct products are defined and discussed briefly. Then follows the introduction of permutation groups and a proof that every finite group can be represented as a regular permutation group. The notions of transitivity and primitivity are not mentioned. The chapter closes with a derivation of the principal theorems about finite Abelian groups—existence of a basis, invariants, and related topics.

Chapter III. Algebraic Fields. The intuitive point of view is used to a considerable extent in this chapter, a more abstract treatment of fields being postponed until later. Quadratic and cubic fields are studied in some detail before more general algebraic fields are introduced. The Galois group of an equation is introduced as a group of automorphisms of its root field, and is then shown to be isomorphic to a group of permutations of the roots of the given equation. There follows a brief discussion of the Galois theory, including a derivation of Galois' criterion for solvability by means of radicals.

Chapter IV. Integral Algebraic Domains. Quadratic integral domains are studied in detail as a background for domains of higher degree. Unique factorization into primes is shown to exist in some quadratic domains but not in others, thus leading to the concept of *ideal* and the unique factorization of ideals into prime ideals. The actual determination of all prime ideals in a quadratic domain is carried out. Next the integral domain consisting of all algebraic integers in an algebraic field of degree n is considered, and ideals are introduced. The arithmetic of ideals is again carried far enough to obtain the unique factorization theorem.

Chapter V. Rings and Fields. This chapter opens with abstract definitions of *ring*, *non-commutative field*, and *integral domain*. Incidentally, the reviewer is a bit unhappy at the use of a terminology according to which a commutative field is a special case of a non-commutative field. A *principal ideal ring* is defined as an integral domain having certain additional properties. Later on, when ideals in general rings have been introduced, it is shown that a principal ideal ring is an integral domain in which every ideal is principal, thus justifying the earlier terminology. The abstract construction of the quotient field of an integral domain is carried through in detail. Then follows the definition of a *prime field* and the determination of all prime fields. Polynominal rings are introduced abstractly, and various questions of factorization are discussed. Ideals are defined, and the notion of *residue class ring* is introduced, thus leading to the determination of all rings homomorphic to a given ring. A satisfactory treatment of algebraic extensions of a given field can now be carried through without use of the fundamental theorem of algebra. Galois (finite) fields are introduced and fundamental results, including a determination of the group of automorphisms, are derived. The concluding section considers the field $F(\lambda)$ obtained from F by a transcendental extension and, in particular, characterizes all automorphisms of $F(\lambda)$ leaving elements of F fixed.

Chapter VI. Perfect Fields. If a and b are elements of a field F, a function $\phi(a)$ is a *valuation* for F if $\phi(a)$ is a positive number or 0 of some ordered field, subject to the further conditions:

$$\phi(a) > 0 \text{ for } a \neq 0,\ \phi(0) = 0,\ \phi(ab) = \phi(a)\phi(b),\ \phi(a+b) \leqq \phi(a) + \phi(b).$$

The real field is obtained as the field of regular sequences of rational numbers, with the ordinary absolute value as the valuation used in defining regularity. This field is then shown to be *perfect* with respect to this valuation, that is, it is incapable of further extension by means of regular sequences of real numbers. The field of complex numbers is obtained by a simple algebraic extension of the real field, and a proof is given that every algebraic equation with complex coefficients has a complex root. Another valuation of the rational field is obtained as follows. Let p be a fixed rational prime. If $a \neq 0$ is a rational number, it is uniquely expressible in the form $(r/s)p^n$, where

r and s are integers prime to p. Then $\phi(0)=0$, $\phi(a)=p^{-n}$ is a valuation of the rational field. The Hensel p-adic number field Ωp is obtained by use of this valuation in the same way in which the reals are obtained from the rationals by use of the absolute value. There follows a series representation of numbers of Ωp and a characterization of the rational numbers in Ωp as those elements with periodic series. It is also shown that a p-adic extension of an algebraic field can always be obtained with a prime ideal playing a rôle analogous to that of the prime p in the case of the rational field.

Chapter VII. Matrices. After a preliminary study of linear systems, the author introduces the concepts of *array* and *determinant* of a square array. A *matrix* is defined as an element of a total matrix algebra. After some preliminary results, the Hamilton-Cayley theorem is obtained, and also a characterization of the minimum equation of a matrix. Some ten pages are devoted to a study of matrices over a principal ideal ring, including such topics as greatest common right (left) divisor, elementary divisors and normal form. The familiar rational canonical form of a matrix over a field is then obtained, as well as the Jordan form for a matrix over the complex field. The regular representations, by matrices, of a finite algebraic field $F(\rho)$ over F are given as an application of the theory of matrices, and probably also as an introduction to the more general representation theory of the next chapter. After a discussion of polynomials in a single matrix, the chapter concludes with a brief treatment of *direct products* of matrices.

Chapter VIII. Linear Associative Algebras. As a concrete introduction to the general theory, the author begins with an unusually detailed study of quaternions, including some new results, due to C. G. Latimer, on quaternion ideals and factorization. Then follows a brief discussion of division algebras, generalized quaternions and cyclic algebras. Linear algebras with unit element are presented in some detail before those without unit element. Further topics treated briefly are regular representations, direct sums, direct products, ideals and the radical of an algebra. General structure theorems are not proved although the principal ones are stated at the end of the chapter.

There are numerous exercises, several sets in each chapter, which give the student an opportunity to test his grasp of the subject, as well as furnishing additional concrete illustrations of the general theories. The author states that students should work all the problems, as they are designed to supplement the rest of the book in an essential way. At the end of each chapter there occurs a short and well-chosen list of suggested texts for further reading. At the end of the book there are tables of Greek and German letters, and a list of some of the more important symbols used in the text.

On the whole the book is well written and the typography excellent. The reviewer noticed a few misprints and even an occasional slip. However, these are mostly of a minor nature and should cause little or no confusion to the student. The book is a valuable and timely addition to the available texts on algebra.

NEAL H. MCCOY

Gap and Density Theorems. By Norman Levinson. (American Mathematical Society Colloquium Publications, vol. 26.) New York, American Mathematical Society, 1940. 8+246 pp. $4.00.

In this book the author confines himself to a detailed study of a few salient topics in gap and density theory; he does not attempt to write a systematic treatise on the subject. The book is in form essentially a collection of research papers; it achieves unity principally through the author's repeated application of similar methods to a variety of problems. Most of the contributions to gap and density theory contained in the book are the author's own work, some of the most remarkable of them being published here for the first time. The principal topics treated are, on the one hand, the influence of the distribution of a sequence of numbers $\{\lambda_n\}$ on the closure properties of the sequence $\{e^{i\lambda_n x}\}$, and the closely related topic of the influence of the distribution of the λ_n on the growth of analytic functions which vanish or are otherwise restricted at points $z=\lambda_n$; and, on the other hand, general Tauberian theorems involving gap conditions. Among the topics not treated are, for example, the Paley-Wiener theory of "pseudoperiodic" functions, and Bochner's generalizations of it. The extensive "classical" theory connecting gap or density properties of a sequence $\{\lambda_n\}$ with the position of the singularities of the function having the Dirichlet series $\sum a_n e^{-\lambda_n x}$ is represented by one theorem. The author expects his readers to be familiar with approximately the amount of information contained in Titchmarsh's *Theory of Functions*. Familiarity with the Colloquium Publication of Paley and Wiener is not prerequisite, but would be advantageous for a reader. The author collects in an appendix the auxiliary theorems which he most frequently uses. His principal tools are such things as Jensen's theorem, Carleman's theorem which is its analogue for a half-plane, Phragmén-Lindelöf theorems, and the L^2 theory of Fourier transforms; these he combines in ingenious and often unexpected ways.