to (2). This same approach has been used by Lotka in his work on renewal theory.

The remaining four chapters deal with special problems such as determination of marriage rates, fertility rates and other indices and measures of natural increases in a population; determination of number of maternal, paternal and complete orphans; calculation of probabilities of extinction of a given line of descent and similar problems. The solution of the last problem, which has also been treated by R. A. Fisher, is extremely ingen'ous. A generation function $f(x)=\sum_{i=0}^{k}\pi_i x^i$ $(\sum_{i=0}^{k}\pi_i=1)$ is set up, where π_i is the probability that a male will have exactly i sons. $f(x)$ has many remarkable properties which enable the author to deal with the problem of descendance. For example $\partial f/\partial x]_{x=0}$ yields the average number of sons per male; the coefficient of x^s in $f^r(x)$ yields the probability that r males will have a total of s sons; the coefficient of x^s in $\pi_r f^r(x)$ which is a general term in $f(f(x))=f_2(x)$ say, is the probability that a man will have r sons and s grandsons. Similar interpretations can be placed on coefficients of x in the iterated function

$$f\{f(f[\cdot\cdot\cdot])\} = f_n(x).$$

The second monograph is well illustrated throughout by applications of the theory to actual census data taken from the United States, England, France, Germany, and several other countries. Numerous charts and tables are given for comparing theory with facts. The monograph is an excellent account of results which have been obtained during the past quarter of a century in the theory of population dynamics. Most of the results are due to the author himself. Lotka has shown a great deal of ingenuity in formulating the problems mathematically and in reaching practical solutions of the problems. Those interested in applied mathematics in the field of biology will find these monographs well worth reading.

S. S. WILKS

The Variate Difference Method. By Gerhard Tintner. (Cowles Commission for Research in Economics Monograph, no. 5). Bloomington, Indiana, Principia Press, 1940. 13+175 pp.

Various mathematical statistical methods have been proposed during recent years in attempts to describe, analyze and interpret economic time series. Regression analysis and its extension to harmonic analysis, moving averages, and the variate difference method are some of the techniques which have been used. The fundamental concept adopted in these approaches is that each element w_i ($i=1, 2, \cdots, N$) in a time series is assumed to consist of a sum m_i+x_i where m_i is the "true economic component" and x_i is the "random component." The main purpose of the statistical analysis according to these methods is to estimate and describe these two components. There are, however, other approaches which use different conceptual models. As its title indicates, the present book presents a treatment (in ten chapters) of the variate difference method, although the author takes advantage of several opportunities to discuss other methods by way of comparison.

In the variate difference method it is assumed that the m_i are "smooth" in the sense that the differences $\Delta^{(K)}(m_i)$ of the m_i of some (finite) order K vanish, while the x_i are independent random variables. Under these assumptions the main problem as discussed by Tintner is that of deciding, on the basis of the w_i and probability theory, for what value of K, $\Delta^{(K)}(m_i)=0$. The problem is therefore one of studying the probability theory of $\Delta^{(K)}(w_i)$ ($K=0, 1, 2, \cdots$) and making suitable significance tests. For large values of N, Tintner follows rather closely the work of Oskar Anderson, which consists in comparing the difference between the variances of two consecutive series of differences, say $\Delta^{(K)}(w_i)$ and $\Delta^{(K+1)}(w_i)$, with the variance of the difference under the assumption that $\Delta^{(K)}(m_i)=0$. The kurtosis of $\Delta^{(K)}(w_i)$ (i.e., $\mu_4'-3\sigma^2$) is also considered in this analysis.

For the case of small N, Tintner proposes a significance test for testing the hypothesis that $\Delta^{(K)}(m_i)=0$, using the two series $\Delta^{(K)}(w_i)$ and $\Delta^{(K+1)}(w_i)$. If the original x_i are assumed to be normally and independently distributed with variance σ^2, then it can be shown that $\Delta^{(K)}(w_i)$ are normally distributed with zero means and variance $C_{2K,K}\sigma^2$. Under the same assumptions, the terms $j+r(2K+3)$ ($r=0, 1, 2, 3, \cdots$) and terms, $j+K+1+r(2K+3)$ ($r=0, 1, 2, 3, \cdots$) will all be normally and independently distributed. Hence σ^2 can be estimated independently from the two series thus making it possible to set up an F-test. Such an F-test can be set up for $j=1, 2, 3, \cdots, 2K+3$.

The significance test based on this method of selection, while mathematically exact under the assumptions made, is very simple but not very efficient, as the author points out, in the sense that full use is not made of the data. A more efficient test would undoubtedly be very complicated.

This book is written primarily for the economic statistician who may wish to apply the variate difference method to his own economic time series. Tintner illustrates the routine application of the method in great detail, using American Wheat-Flour prices for the period

1890–1937 as his data. The author gives a large number of tables of weights and coefficients of various kinds, critical ratios of two sums of squares of differences, formulae for selections of items in various pairs of consecutive differences which would be independent, etc., to facilitate the application of the method.

A considerable amount of historical material on the variate difference method is given. One chapter is devoted to the application of Sheppard's smoothing formulae and serial correlation, and the connetion between the variate difference method and these methods is pointed out.

Several appendices are given: Appendix I is a summary of computation formulae used in the various chapters; Appendix II is devoted to the mathematics underlying the various formulae which are used; the remaining four appendices deal with special topics such as seasonal variation, normality of the random element, etc. Author and subject matter indices are provided.

The question as to whether the variate difference method is superior to other methods of time series analysis is largely a matter of opinion. Tintner has given a very good account of the method, although it appears to the reviewer that he did not concern himself enough with the problem of estimating the m_i after it had been decided for which value of K the value of $\Delta^{(K)}(m_i)=0$. He approached the problem of determining the m_i by using the Sheppard smoothing formulae, but this, of course, is essentially another method of time series analysis.

Tintner's book is an interesting contribution to the literature of time series analysis. It is well documented by references and footnotes and reflects a great deal of work on the part of the author.

S. S. WILKS

Tensor Analysis of Networks. By Gabriel Kron. New York, Wiley, 1939. 24+635 pp. $7.50.

In the introduction to this book, the author states: "It is emphasized that this book is not written by a mathematician and is not written for mathematicians. This book is written by an engineer for engineers who are interested in learning an *organized* method of attack to analyze and synthesize electrical networks."

The first two chapters provide an extremely detailed account of the tensor and matrix notation, of the multiplication of matrices and describes the impressed voltages, currents and self- and mutual impedances of a network as components of tensors e_α, i^α and $Z_{\alpha\beta}$, re-