ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

ALGEBRA

115. J. H. Bell: *Topics related to the factorization of matrices.*

This paper deals with two problems that are of particular interest in the solution of the unilateral matrix equation. Let $\mathfrak{D}$ be a quasi-field and $\mathfrak{D}[\lambda]$ the polynomial ring of $\mathfrak{D}$, where λ is a commutative indeterminate. The first problem is: Under what conditions is the matrix A with elements in $\mathfrak{D}[\lambda]$ the left associate of a matrix which is proper of degree k in λ? A necessary and sufficient condition is obtained. The proof may be shortened in the case that $\mathfrak{D}$ is a field $\mathfrak{F}$. The second problem, which is considered only in the case of a field $\mathfrak{F}$, is: Given two nonsingular matrices A and B with elements in $\mathfrak{F}[\lambda]$; if A and B are not left associates, what other proper right divisors of the least common left multiple of A and B exist which are not left associates of A or B? Various results are obtained for the second problem. These results may be used, with the result obtained in solving the first problem, to show the existence of families of solutions of the unilateral matrix equation, if two solutions satisfying certain conditions are known to exist. (Received January 23, 1941.)

116. A. H. Clifford: *Semigroups admitting relative inverses.*

A semigroup is a system S closed under an associative binary operation: $(ab)c = a(bc)$. S is said to admit relative inverses if for each a in S there exists an element e of S such that (1) $ea=ae=a$, and (2) $aa'=a'a=e$ for some a' in S. The "gross structure" of such an S is determined by showing that S is the class sum of mutually disjoint semigroups S_α of known structure (A. Suschkewitsch, D. Rees) which can be arranged in a semi-lattice P such that the set-product $S_\alpha S_\beta$ of any two of them is wholly contained in the "cross-cut" S_γ of S_α and S_β in P. If the idempotent elements of S commute with each other, then the S_α are groups, and the structure of S can be given completely. (Received January 13, 1941.)

117. A. H. Clifford and Saunders MacLane: *Factor sets of a group in its abstract unit group.*

This paper is an investigation of the structure of the set of group extensions of a certain abstract unit group. This group is one which arises in class field theory and has as group of operators a certain finite group Γ. If Γ is solvable, it is shown by an explicit reduction that the group of group extensions is isomorphic to Schur's multiplicator for the group Γ. (Received January 28, 1941.)

118. Marshall Hall: *A problem in partitions.*

Non-void subsets $a_1, \cdots, a_n$ of a set $S(x_1, \cdots, x_n)$ determine a partition matrix

(a_{ij}) in which $a_{ij}=0$ or 1 according as a_i and a_j overlap or not. The question has arisen as to whether every n^2 matrix (a_{ij}) in which $a_{ii}=1$, $a_{ij}=a_{ji}=0$ or 1 is the partition matrix of n subsets of n objects. This is shown to be true for $n=1, 2, 3, 4$, but not for $n \geqq 5$. It is shown that for $n \geqq 5$ every such matrix is a partition matrix but may require as many as $n^2/4$ (n even) or $(n^2-1)/4$ (n odd) objects in S. These values are "best possible" and it is shown that a matrix requiring the maximum number of objects is unique to within an equivalence amounting to renumbering $a_1, \cdots, a_n$. (Received January 24, 1941.)

119. E. R. Kolchin: *On the basis theorem for differential systems.* Preliminary report.

The basis theorem of J. F. Ritt and H. W. Raudenbush for systems of differential polynomials is proved by a new method making no use of ascending sets, but using the axiom of choice. The new proof justifies its existence by the greater generality of its results. For example, in addition to the systems for which the theorem has already been proved, systems of forms with coefficients in a differential field of characteristic zero, the theorem is proved for systems of forms with coefficients in the ring of integers, coefficients in the ring of integral polynomials in x, and coefficients in the field of integers modulo a prime. (Received January 25, 1941.)

120. Howard Levi: *On the ideal theory and structure of differential polynomials.*

In the first part of this paper the differential ideal Σ generated by the form y^p is investigated. It is understood that p is a positive integer greater than unity, and that the underlying ring is that of all differential polynomials in the unknown y whose coefficients are in some differential domain of integrity which contains the rational numbers. Necessary and sufficient conditions for the membership of a form in Σ are obtained. These results are used to show that the criteria (sufficient) of J. F. Ritt for the essentiality of manifolds (Annals of Mathematics, (2), vol. 37 (1936), pp. 556–560, and American Journal of Mathematics, vol. 60 (1938), pp. 5–14) are valid in the abstract case. Extensions are made to cases not discussed by him. Finally the differential ideal generated by the form uv in the ring of forms in the unknowns u and v is examined. Necessary and sufficient conditions for membership in this ideal are obtained. In particular it is shown that for all positive integers r and s the form $u^r v_r^s$ is not in the ideal (v_r being the rth derivative of v). This implies that the ideal in question has no representation as the intersection of differential ideals. (Received January 24, 1941.)

121. Oystein Ore: *Theory of monomial groups.*

If $\{x_i\}$ denotes a set of variables and H some arbitrary group, then the set of all linear transformations $x_i \rightarrow h_i x_j$ where the factors h_i belong to H, form a group called the complete monomial group. The present paper contains a study of these groups and gives among other results the determination of all normal subgroups and the group of automorphisms. (Received January 23, 1941.)

122. Oystein Ore: *Transformation of sets.*

The set of all one-to-one correspondences of a set S to itself form a group Σ_s. It is shown that for an arbitrary infinite set this group is complete, that is, all automorphisms are inner automorphisms. (Received January 23, 1941.)

123. P. C. Rosenbloom: *Post algebras: I. Postulates and general properties.*

Post algebras are the algebras which bear the same relation to the n-valued logics defined by Post (American Journal of Mathematics, vol. 43 (1921), pp. 180–185) as Boolean algebras bear to the usual 2-valued logic. They are investigated here purely from the algebraic standpoint with no consideration of their interpretation as logic. The first postulate sets for these systems are introduced and the most important general properties are deduced. A fundamental theorem analogous to the Boolean expansion of functions in normal form is proved. The definition of "prime elements" is analogous to Huntington's definition for the Boolean algebras. "Powers of primes" are also defined. A theorem analogous to the fundamental theorem of arithmetic is proved to the effect that "in a Post algebra with a finite number of elements, every element is uniquely factorable into a product of powers of primes, disregarding order and repetition of factors." An arithmetic interpretation generalizing Sheffer's "Boolean numbers" is given. Several unsolved problems are discussed. (Received January 17, 1941.)

124. Ernst Snapper: *Structure of linear sets.*

It is shown that the linear sets of a vector space of arbitrary dimension over an integral domain in which every ideal has a finite basis admit a Noether decomposition into "primary" linear sets. The "associated prime ideals" of the largest primary components are uniquely determined invariants of the linear set. The proofs are based on definitions of "quotient ideal" of a linear set by a linear set, of "quotient linear set" of a linear set by an ideal, and of "product linear set" of an ideal and a linear set. The quotient ideal of a linear set by the whole space is called its "essential scalar ideal" and is fundamental in the definition of primary linear set. The radical of the essential scalar ideal of a primary linear set is prime and is called its associated prime ideal. The "isolated component linear sets" are uniquely determined by their corresponding prime ideals and the theory becomes the ordinary ideal theory in the case of dimension one. Also, this investigation gives rise to the notions of scalar ideal, almost-primarity, radical and essential radical, closed set, and dense set. (Received January 14, 1941.)

ANALYSIS

125. R. P. Agnew: *On methods of summability and mass functions determined by hypergeometric coefficients.*

Let α, β, γ be complex constants and $\gamma \neq 0, -1, -2, \cdots$. Let $\lambda_n(\alpha, \beta, \gamma)$, $n=0, 1, 2, \cdots$, be the coefficients in the power series expansion $\sum \lambda_n z^n$ of the hypergeometric function $F(\alpha, \beta, \gamma; z)$. Let $(H, \alpha, \beta, \gamma)$ be the Hurwitz-Silverman-Hausdorff method of summability generated by the sequence $\lambda_n(\alpha, \beta\ \gamma)$. (See Garabedian and Wall, Transactions of this Society, vol. 48 (1940), pp. 195–201.) Let C_r denote the Cesàro method of order r. For certain ranges of the parameters it is shown that $(H, \alpha, \beta, \gamma) = C_{\alpha-1}^{-1} C_{\beta-1}^{-1} C_{\gamma-1}$ and that $(H, \alpha, \beta, \gamma)$ is equivalent to $C_{\gamma-\alpha-\beta+1}$. These results determine conditions under which $(H, \alpha, \beta, \gamma)$ is regular and $\lambda_n(\alpha, \beta, \gamma)$ is the moment sequence of a regular mass function. (Received December 31, 1940.)

126. E. F. Beckenbach: *On almost subharmonic functions.*

It is shown that certain integral inequalities which, in the case of continuous functions, are known to characterize subharmonic functions and functions whose loga-