A History of Geometrical Methods. By J. L. Coolidge. Oxford, Clarendon Press, 1940. 18+452 pp. $10.00.

Notwithstanding the rich literature on geometry, there has been no comprehensive synopsis of the subject for a long time. The author has set himself the task of presenting in outline the development of geometry from the beginnings to the present, emphasizing in particular the various methods employed, taking for a general model Chasles's *Aperçue Historique*, which first appeared in 1837. The field is so large that not all material could be considered; selections must be made, and no fixed criterion can be set down for eligibility.

The book is divided into three parts: synthetic, algebraic, and differential. It is explicitly stated that topology is excluded from consideration. The first part covers 116 pages. It includes a sketchy summary of practically everything preceding Descartes, all of synthetic projective geometry, non-euclidean geometry in the narrower classic sense, and a partial discussion of descriptive geometry; axonometry is not mentioned, and the treatment of perspective is very brief. Pohlke's theorem is not mentioned. The substance of this part is largely considered as a completed edifice.

The second part, algebraic geometry, covers 200 pages. It is the field of most of the author's own creative activity; many topics are referred to his extensive writings for fuller discussion. After pointing out various usages of coordinates in earlier writings, the classic period of Fermat and Descartes is given in some detail, followed by a discussion of abridged notation, change of space element and extension to higher dimensions. This in turn is followed by a short description of the Clebsch-Aronhold symbolic notation, minimal coordinates, of elliptic coordinates, pentaspherical coordinates, and other systems.

Unusual care is devoted to the study of enumerative geometry, particularly to the Schroeter calculus. The opinion is suggested that this subject has provided more errors in the literature of the subject of algebraic geometry than any other. After a stormy and acrimonious history, the principles of the discipline are now fully established, but in order to apply them properly, the operator must know how to count. The closely allied theory of correspondence between points of associated algebraic loci is discussed from the same point of view.

The chapter on birational geometry furnishes a good introduction to the geometry on algebraic curves and surfaces; rather more emphasis is put on the transcendental treatment than the history of the literature of the subject warrants. The treatment of the uses of higher spaces is well written, but too brief to be of maximum usefulness, except possibly the paragraph on quaternions. The chapter on transformations begins with linear ones (both collineations and correlations), then quadratic ones and the general Cremona transformation. In the plane, this is practically complete. Like synthetic geometry, this subject involves a number of indispensable tools, the use of which must be thoroughly mastered for further progress, without the hope of contributing much from this section that is new. On page 288 is the remark that Mlodziejowski computed the tables of plane Cremona transformations to order 21. Without debating the real merits of this work, the reviewer inclines to the opinion that the Naples dissertation (1909) of Marazzo should also be mentioned, as it provides the tables of orders up to 23.

After a careful and correct discussion of Hudson's fundamental theorem that in space of more than two dimensions a theory of composition analogous to Noether's theorem for the plane does not exist, the author dismissed the theory in more dimensions with the implication that there isn't much there. In the sense of results, this is correct. The author ascribes this state of affairs to the greater difficulty and to the lack of general methods. What greater incentive is needed? During the last few years the question of the existence of irrational involutions of order two in three way space has been discussed in over a hundred papers, and that of a possible new approach to the question of composition has a literature that is nearly as large. It is pointed out that the group concept will probably play an important part in further advances along this line.

The third and last part of the book, differential geometry, covers 100 pages. After a brief glance at the early writers, including Euler and Monge, a detailed exposition of the methods of Cauchy follows, then the use of intrinsic methods and of movable axes. A full analysis of the work of Gauss is given; it plays an important part in much of the later metric differential geometry. Geodesic lines on a general surface are treated, followed by a summary of the essential property of minimum surfaces. A brief discussion of differential properties of line congruences is then given.

The chapter on projective differential geometry compares the methods of Wilczynski, Fubini, Cartan, and their followers. The name Halphen is not mentioned in this connection, but he was more truly a founder than the others. The range of problems to which these methods are applicable is sharply limited, but within these limitations, important results are obtained with ease and elegance.

The last chapter, on absolute differential geometry, is devoted largely to an explanation of the notation employed. This is not entirely consistent, and the pace is so rapid that its usefulness remains in doubt. Riemannian geometry, parallelism and the geometry of paths are all touched upon.

The book is provided with a subject index and an extensive bibliographical one. The latter includes (most of) the names cited, including the title and a reference to its source, and in many cases the date of birth and of death of those cited. This is a valuable list; its compilation is a difficult and often a thankless task. A real source of confusion arises in some cases in which the information is not complete, especially when the person cited was born after the middle of the nineteenth century. A special symbol to indicate that the entries were not complete in such cases would have avoided the ambiguity. Numerous slips or actual errors were noticed. Six proper names are misspelled; for homaloid the spelling homoloid is everywhere used. In the formula at bottom of page 221 for i read $i-1$. A real source of confusion arises on page 220 line 10 up. The genus defined is not the geometric genus, but the number p' in the author's notation. The discussion on the following page also refers to p'. This is what Noether called the Curvengeschlecht.

On page 208 is a footnote concerning the origin of the transformation connecting adjoints of plane curves, and hyperspace. Compare Cayley: *On the transforms of curves*, Proceedings of the London Mathematical Society, vol. 1 (1865). The formula $I+p-\sigma=12p_a+9$, in which I is the invariant of Zuethen-Segre and σ the number of exceptional curves in the system, is discussed in the footnote page 227. Compare Noether, Mathematische Annalen, vol. 8 (1875), pp. 495–528.

Professor Coolidge has put a great deal of careful thought in the preparation of this book. A number of concepts have been correctly explained, which have been sources of confusion in the minds of many, especially those interested primarily in other branches of mathematics. The book will serve a real purpose.

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Festschrift Rudolph Fueter zur Vollendung seines sechzigsten Altersjahres, 30 VI. 1940. Zürich, Naturforschende Gesellschaft, 1940. 231 pp.

This volume comprises articles by Oystein Ore, Henri Lebesgue, M. Plancherel, N. Tschebotaröw, Paul Montel, W. Scherrer, L. J. Mordell, Francesco Severi, T. Carleman, E. Hecke, H. S. Vandiver, R. Wavre, H. Brandt, C. Carathéodory, Heinrich Jecklin, Eugenio G. Togliatti, Alfred Kienast, Ernst Trost, Ludwig Bieberbach, J. J.