*Reelle Funktionen*, Part I. By C. Carathéodory. Leipzig, Teubner, 1939. 6+182 pages. RM 11.20.

This volume is Part I of a projected three volume treatise on real functions to be distributed under the following titles: Part I, Numbers, Point Sets, Functions; Part II, Theory of Measure and The Integral; Part III, as yet without a title, but presumed to treat of derivatives and applications.

These volumes will replace the author's one volume, *Vorlesungen Über Reelle Funktionen*, first completed in 1917 and later reissued in 1927 with slight changes. It will be recalled that this earlier volume is written in the elegant axiomatic style for which the author is noted. Among its outstanding features are an abstract treatment of measure and a very full treatment of Lebesque integration.

This volume preserves much of the material in the first 230 pages of the older treatise. A considerable rearrangement of material has been made which adds to the attractiveness of the book. But examples are given only when required to show the necessity for the conditions governing certain theorems. The reviewers believe that the addition of illustrative examples would have been appreciated by many readers.

Some of the essential changes and additions will be taken up by chapters.

Chapter I—Real Numbers. Denumerability and non-denumerability treated in Chapter II of the older treatise here follows an axiomatic introduction to real numbers.

Chapter II—The Limit Notion. A discussion of Cantor's diagonal process is added to the older chapter on limits.

Chapter III—Point Sets in Euclidian Space. The treatment of open and closed sets also involves an iteration process of forming closures and interiors. An application is made to boundary theory.

Chapter IV—Normal Covering Sequences and the Theory of Connectedness. A new feature is the treatment of connectedness of arbitrary sets.

Chapter V—Functions. No essential change over the older volume save for omission of a discussion of functions of bounded variation (deferred to Part II) and the inclusion of a section on continuous monotonic functions in which is introduced a transformation function

$$x = \lambda(y) = \frac{1+y+|y|}{2(1+|y|)}$$

which sends the $y$-interval $(-\infty, \infty)$ into the $x$-interval (0, 1). This preserves local limit properties and permits the extension of some theorems on positive bounded functions to more general functions.

Chapter VI—The Distance Function and Applications. The chapter closes with Radó's proof of a theorem on the extension of closed domains of definition of functions. A different proof was given much later in the older volume.

Chapter VII—Sequences of Functions. This chapter contains perhaps the most interesting addition to the corresponding material of the older volume; namely, continuous convergence and normal function families. The theory of continuous convergence is developed in an elegant manner. However, the author might have pointed out the close connection existing between this theory and ordinary continuity of functions. By the simple device of considering the subscript $k$ of the sequence $f_k(P)$ as another coordinate in our number space, continuous convergence of the sequence becomes identical with the existence of a limit at the point $(P_0, \infty)$ of the corresponding function $F(P, k)$ in the extended space. Thus, for example, Theorem V follows immediately when it is noted that the two conditions given there are equivalent to the requirement that the saltus of $F(P, k)$ vanish at $(P_0, \infty)$, while Theorem VII follows directly from the theorem: A function continuous on a bounded and closed set is uniformly continuous on that set.

The final section of the chapter, on normal families of functions, while brief, contains recent results on function sequences of great interest to the advanced student of analysis.

Some readers will find it regrettable that the author retains in this modern work his limitation of the discussion to euclidean spaces, thus leaving out of account the far-reaching extension of real function theory to more general spaces made during the last thirty years; the more so, as much of the theory can be given in this general setting without in any way complicating the reasoning or lengthening the proofs. Bohnenblust's 1938 *Princeton Notes on Real Variables* indicates the possibilities of such a treatment.

Since this volume contains the more elementary material many readers will naturally be more interested in Parts II and III. The known high standards of the author are enough to insure the completed work a prominent place in mathematical literature.

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