

ity of a sequence of functions has yielded new information on one or more of these topics.

At least two decades ago, Montel conjectured that a family of functions $f(z)$ analytic in a region is normal there provided $f(z)$ fails to assume the value zero and $f'(z)$ fails to assume the value unity. This conjecture was discussed at least orally by many mathematicians, and was finally proved in 1935 by Miranda. Miranda had been preceded by Bureau, who obtained some related theorems by the use of the Nevanlinna theory of meromorphic functions but who did not establish Montel's conjecture. Miranda's work was based also on the Nevanlinna theory and on Bureau's results. Valiron subsequently introduced the direct methods that he had previously employed in the study of the theorems of Picard and Borel to obtain wide generalizations of Miranda's theorem.

The fascicle before us is devoted to a systematic exposition of this body of material, together with many new generalizations and complements. There are close connections with the theorems of Picard, Landau, Schottky, and Bloch. The proofs are far more than pure existence proofs, for they involve specific inequalities of a numerical nature on the modulus of the functions $f(z)$.

The treatment is clear, pleasing, suggestive—an admirable exposition of a field of current interest and importance. May there soon be made in this country systematic provision for the encouragement of the writing of similar essays and for their publication!

J. L. WALSH

Complex Variable and Operational Calculus with Technical Applications. By N. W. McLachlan. Cambridge, University Press, 1939. 11+355 pp. \$6.50.

Since the time of Heaviside several books on operational calculus have appeared on the market. The majority of them deal almost exclusively with the electrical applications. This is due, no doubt, to its origin in the pioneer work of Heaviside. With very few exceptions these books treat the subject in a formal and heuristic manner leaving much to be desired from the standpoint of mathematical rigor. Several mathematicians have helped to place the subject on a more substantial basis and thereby extend its usefulness. The author has based his treatment of the subject on the Laplace transform and the Mellin inversion theorem in line with modern developments. Except in a few places a sufficient degree of rigor has been maintained throughout the book to meet the needs of the engineer and applied mathematician.

The book is divided into three parts and an appendix. Part I is a treatment of sections of complex variable theory with a view to applications. The second part deals with the operational calculus. In the third part we find the applications to many types of problems encountered in practice. This is followed by an appendix dealing with some theoretical questions and an extensive bibliography.

The first chapter plunges the reader into the theory of complex variables. No time is wasted before the introduction of singular points. An essential singularity is defined as a pole of infinite order which is rather loose. The principal part of the Laurent expansion provides a clear as well as a rigorous definition of the singularities of a single valued function, but this is postponed until the second chapter. This is an example where suggestive language does not lead to clarity but where rigor does not imply a lack of vigor. A few slight lapses like this one mar an otherwise admirable presentation. Branch points receive a very good treatment from the point of view of their use in the applications. The Cauchy-Riemann test for analyticity is completely left out. No doubt, all the functions arising in applications are analytic, but the reader ought to be provided with this important criterion. Integration is taken up in the second chapter. Cauchy's theorem is stated without proof. The Cauchy integral formula, Taylor's theorem and the Laurent expansion round out the chapter. Integrals indefinitely close together and infinitesimal gaps in the contours provide another example of loose but suggestive language.

A very thorough but not always rigorous treatment of the calculus of residues and contour integration around branch points is given in chapter three. This leads up to the treatment of the Bromwich-Wagner integral in the next chapter. By the use of Jordan's lemma and deformation of the contour several important cases of the Bromwich integral receive a clear and thorough treatment. The author closes the fourth chapter with some very neat methods to obtain approximate values of integrals needed in the sequel.

Three of the most useful functions are the Gamma, Bessel, and Error functions. In the sixth chapter an adequate treatment of these functions is provided as well as a discussion of asymptotic expansions and analytic continuation. When branch points occur the treatment of the Bromwich integral is very fussy and the author has treated it with some care. A short chapter on differentiation under the integral sign and other iterated limit problems closes the first part of the book.

Part II introduces the operational calculus on the basis of the p -multiplied form of the Laplace transform and Mellin inversion

theorem. The conditions under which the inversion is possible are narrow but suitable enough for the purpose at hand. The proof is relegated to the appendix. The term "operational form" is used instead of the more customary word "transform." Although the operational forms of the derivatives of functions with arbitrary initial conditions are derived, very little use is made of them except in the case where the function and all its derivatives vanish when t is zero. The shift operator is neatly applied to obtain Fourier expansions in special cases. A treatment of ordinary differential equations with constant coefficients, Heaviside's unit function, and impulses closes the second part.

The technical problems treated in the third part are too numerous to mention. A feature here is the thorough treatment of several problems with the attainment of numerical results. This is after all the final aim of the technologist. The problems involving partial differential equations are not as clearly stated as those connected with ordinary derivatives.

A final criticism of omission must be made. The central theorem known as the "faltung" or convolution theorem is not even treated as a step-child. This theorem is known to the engineer as Borel's theorem. Under this name it receives only passing attention. The publication of this book is a step in the right direction and the author is to be congratulated for having written a book which will be very valuable to the mathematical technologist.

SAMUEL SASLAW

Théorie Mathématique du Bridge à la Portée de Tous. By Émile Borel and André Chéron. Paris, Gauthier-Villars, 1940. 17 + 392 pp. Fr. 175.

Here is a detailed study of shuffling, card distributions, the finesse, and other points of the game of bridge by an eminent mathematician and an authority on bridge. A survey of the context naturally enough begins with their initial discussion of the shuffling of the deck.

Shuffling of type A is that generally used in which the deck is separated into two approximately equal parts and then dove-tailed together in small alternating packets. Let a sequence be defined as two adjacent cards in an initial order. If the average number of sequences in the deck broken by one operation of A is S and N is the number of such operations, then NS greater than 150 will result in a well shuffled deck. Another test of the adequacy of shuffling is the fact that the mean number of unicolor sequences (two adjacent cards of the same color) is 12. Shuffling of type B : A packet of a_1 cards is