ory the center of the earth's orbit about the sun is taken as the center of the universe. Kepler's innovation moved the center (focus) to the sun itself and gave a mathematical law for the varying velocity of each planet in its orbit. Vital in the computations was new light upon the earth's orbit. In fact, the second law of Kepler that equal areas are swept out in equal times was first demonstrated by Kepler for the earth; the formulation gave the velocity as inversely proportional to the distance from the sun, the "radius-theorem." The final establishment for the orbit of Mars was dependent upon the corrections made possible by the revisions made in the earth's orbit.

In preparing for publication of Kepler's collected works translations proved to be out of the question; the editors have introduced the device of admirable summaries of contents of each volume. In addition there are notes, often giving an indication of the method of work of the author, and the genesis of the ideas; in fact, Kepler himself gives much information along this line, including even material finally rejected. The editors have also included in each volume numerous notes and for the periods concerned available lists of Kepler's correspondence, largely with brief statements of contents of the letters.

In every way the series, *Johannes Kepler Gesammelte Werke*, can be commended as worthy of the great genius of Kepler.

LOUIS C. KARPINSKI

*An Introduction to the Theory of Functions of a Real Variable.* By S. Verblunsky. Oxford, Clarendon Press, 1939. 11+169 pp. $4.25.

This text for students and teachers was written for the special purpose of furnishing a more rigorous and accurate treatment of the elements of the theory of functions of a real variable. It is based on notes of lectures delivered by Verblunsky to students in their first year at the University of Manchester. The subject matter is entirely standard, but the treatment involves much that is new and original—ingenious and elegant proofs for certain theorems and new approaches to some parts of the subject. The dominant feature of the book is the presentation of the subject as a body of deductions from specified hypotheses.

The material treated will be sufficiently indicated by the following list of chapter headings: Chapter I, Number; II, Sets and Functions; III, Convergence; IV, Continuity and the Derivative; V, The Elementary Functions; VI, Primitives; VII, Limits and Higher Derivatives; VIII, Integrals; IX, Series. The elementary character of the book should be noted. There is no treatment of the properties of sets of points, of measure, of Lebesgue integration, or of other modern topics.

The treatment is different or unusual in a number of respects; some of them are the following. (a) In Chapter I there is a set of postulates from which all the results in the book are derived. These postulates do not form a mutually independent set; they were chosen in an effort to make the initial progress of the student rapid and easy. (b) In Chapter I also the principle of induction is stated as a postulate; it is then used constantly, and whenever occasion demands, in proofs throughout the book. (c) Both Heine's and Cauchy's definitions of continuity are given, but the treatment is based largely on the former. Although there is a proof that the two definitions are equivalent, attention is not called to the use in it of the axiom of choice. Two definitions of a continuous function $f(x)$ are given, one for a closed interval (Chapter IV, p. 51) and another for an open interval (Chapter VI, p. 83). Finally, the treatment of continuity is separated from the treatment of limits of functions of a continuous variable, the former being placed first in Chapter IV and the latter afterward in Chapter VII. (d) The treatment of the derivative, placed as it is in Chapter IV before the study of limits in Chapter VII, obviously involves new features. The derivative of $f(x)$ at $x=\xi$ is shown to be the value $\phi(\xi)$ of an auxiliary function $\phi(x)$ which is continuous at $x=\xi$. There follow elegant derivations of all the standard properties of the derivative. (e) The treatment of the elementary functions (exponential, logarithmic, trigonometric, hyperbolic) in the short space of fourteen pages in Chapter V is a gem. (f) The introduction of an equivalence relation indicates the care used in the treatment of primitives in Chapter VI. The familiar theorem now reads: a primitive of the sum of two functions is equivalent (not *equal*!) to the sum of primitives of these functions. (g) The integral treated in Chapter VIII is essentially a special case of the Perron integral. It is quite general and integrates with equal ease bounded functions and certain functions which become infinite at a finite number of points. It is shown that for a continuous function it is possible to approximate to the integral by the Riemann sums; in general, however, there is no treatment of integrals as the limits of sums. (h) The book contains a large number of examples. Some illustrate definitions and theorems, and others are additional propositions which are essential for later developments. Most of the examples are accompanied by proofs.

To this reviewer it seems that Verblunsky has succeeded admirably in his effort to write a rigorous and accurate introduction to the theory of functions of a real variable. Furthermore, the literature has been enriched permanently by original treatments of certain topics and the elegant proofs of numerous theorems which he has contributed. At the same time, the book has certain characteristics which this reviewer finds unfortunate in a text for beginners. They are these. (a) The subject has been completely divorced from its intuitive background and also from its historical development. The book contains no figures, and there is no indication of the origin of any of the ideas involved or of their applications. There are no references to the history of the subject. Even the names of mathematicians have been omitted; those of Rolle and MacLaurin are the only ones this reviewer could find. (b) There is an authoritative air of finality about the book. There is no suggestion that other treatments can be given for many of the topics, or that interesting extensions and generalizations are possible in some cases. A book with these characteristics is not likely to offer much *inspiration* to a beginning student. Nevertheless, it is a valuable text for superior students who are reading independently and for classes of an inspiring lecturer.

The printing has been done with an accuracy worthy of the author's success in producing a book of high mathematical quality. The reviewer noticed only three typographical errors, all of which were small and obvious; no one would cause confusion even to the beginning student. The author and printer are to be congratulated on producing an excellent mathematical text.

G. Baley Price