analysis from the standpoint of the applications to geometry, mechanics, and physics. In the early revisions of the text Appell made notable changes with a view to strengthening the preparation for the study of mechanics and physics. The fourth edition appeared in 1921 and, in addition to representing the course given at l'École Central, it included certain material which figured in the program of admission; the reader, however, was assumed to have a knowledge of the exponential and logarithmic functions, elementary differential calculus, complex numbers, and the elements of analytic geometry.

In presenting the first volume of the new edition, Georges Valiron has made numerous changes not only in the choice and order of the subject matter but also in the method of treatment. He reminds us that the book is not intended to be a mathematical encyclopedia, but rather a course of instruction which is progressively more difficult and introduces new notions only as they are needed. For example, complex numbers are not presented until very late in the course when the advantage of their use in certain problems is readily appreciated.

The first three chapters, which have been added by Valiron, serve as a geometric introduction and are based on lectures commonly given at the beginning of the course in mechanics at the Sorbonne. In these eighty pages the author introduces the vector notation and then develops the elements of plane and solid analytic geometry. Although Valiron's treatment is more extensive, it reminds one somewhat of Appell's *Éléments de la Théorie des Vecteurs et de la Géométrie Analytique.*

Beginning with the fourth chapter the new edition develops the present program of analysis and geometry for the certificate in general mathematics of the Faculty of Sciences of Paris. This program has been in use since 1931 and it preserves the spirit of Appell's work even though the general plan has been somewhat altered. The scope of this portion of the book is indicated by the chapter headings: 4. Fonctions d'une variable. Limites. Continuité. 5. Fonctions dérivables. 6. Fonctions primitives. Intégrales. Différentielles. 7. Fonctions exponentielle et logarithmique. 8. Méthodes d'intégration. 9. Intégrales définies dont une limite est infinie où portant sur une fonction non bornée. 10. Fonctions de plusieurs variables. 11. Courbes planes ou gauches. Courbure. Enveloppes. 12. Étude des courbes en coordonnées polaires. 13. Intégrales curvilignes. Calcul des aires et des volumes.

A comparison with the fourth edition shows that Valiron has rearranged the subject matter considerably. There are frequent additions; in particular, the chapter on exponential and logarithmic functions is new. Vector methods have simplified many of the discussions; this is especially noticeable in the study of plane and space curves. In this first volume very little use is made of infinite series. Numerous well-chosen examples from the fields of geometry and physics are solved in the text and much emphasis is placed on graphic representation and methods of approximation. The figures are numerous and very well drawn. The text is attractively printed and the number of typographical errors noted was small.

C. H. Yeaton

The Theory of Linear Operators from the Standpoint of Differential Equations of Infinite Order. By H. T. Davis. Bloomington, Indiana, Principia 1936. 14+628 pp.

List of contents: 1. Linear operators. 2. Particular operators. 3. The theory of linear systems of equations. 4. Operational multiplication and inversion. 5. Grades defined by special operators. 6. Differential equations of infinite order with constant coefficients. 7. Linear systems of differential equations of infinite order with constant coefficients—the Heaviside calculus. 8. The Laplace differential equation of infinite order. 9. The generalized Euler differential equation of infinite order. 10. Differential operators of infinite order of Fuchsian type—infinite systems. 11. Integral equations of infinite order. 12. The theory of spectra. Bibliography.

Professor Davis has written a number of papers on differential equations of infinite order. These papers he has now expanded into a voluminous treatise on linear operators. His own investigations form the core of Chapters 2 and 4 to 11 of the present book though he has incorporated much material from the work of other scientists. The reader will find much of stimulating interest in this treatise. The references are unusually complete and the reader who wants to learn something about the history of operational calculus, especially on the formal side, will find this monograph indispensable as a source book. As a matter of fact, one can get some information about almost any mathematical question by referring to the subject index of this book. Sometimes the information is excellent, a case in point being the treatment of Hadamard's determinant theorem on pp. 116–123 with its rich bibliography, but far too often the wording is ambiguous and sometimes misleading. It is a decided weakness of the book that the treatment is spread too thin over too vast an area. The author is inclined to include in the text excerpts from the literature, long quotations, theorems frequently stated without proper background, etc. The book would have gained had this material been severely pruned.

These are criticisms of detail. There are larger issues at stake. As is apparent from the title of the book, the author treats linear operators from the standpoint of differential equations of infinite order. It is quite natural that he should wish the power of this method to appear as strong as possible, but the claims that he makes for it strike the reviewer as insufficiently supported by the evidence presented in the book. His basic operation is differentiation, $z=d/dx$, and practically all his operators are power series in z with coefficients which are analytic in x. The natural domain for such an operator is the field of analytic functions, and, frequently, the set must be narrowed down to entire functions of exponential type. Now it is well known that some differential operators of infinite order coincide in their proper domain of definition with operators having a much wider range of existence. In other words, the operator admits of an extension. The simplest case in point is the operator exp (hz) which, whenever it has a sense, transforms $f(x)$ into $f(x+h)$.

As far as I can see the author supports his claims of having a broad and powerful approach to the operational calculus upon three theses, all highly interesting. The first is a principle of extension: every (respectable) differential operator admits of an extension outside the domain of analytic functions. The second is a special case of a principle of the permanence of functional equations which could be formulated as follows. If the linear equation $y=Ux$ possesses an inverse $x=U^{-1}y$ when y is suitably restricted and if this inverse transformation possesses an extension outside of its original domain of definition, then the extension is still an inverse of the original equation. The third is a related principle of formal invariance. If an inversion formula, derived under restrictive hypotheses on the operator, remains meaningful when these assumptions are removed, then the formula also remains an inversion formula for the general operator.

I do not think that the author has clearly formulated these principles anywhere in his book, but they underlie most of his work. I have no objections against these principles as working hypotheses, the results so obtained being verified directly, or better still, as strictly formulated and rigorously proved theorems. When in Chapter 11, the author sets out to derive the theories of the equations of Volterra and of Fredholm from differential operator theory, he is really invoking the second and third of these principles, but in the absence of proofs of these principles, the value of this approach must be held in abeyance. That the extension principle is valid in special instances is documented by numerous important examples, but the justification in general cases is missing. It is impossible to overlook these defects in a book which otherwise has many good features.

EINAR HILLE

An Introduction to Riemannian Geometry and the Tensor Calculus. By C. E. Weatherburn. Cambridge, University Press; New York, Macmillan, 1938. 11+191 pp.

In the author's words this is "a book which will bridge the gap between the differential geometry of euclidean space and the more advanced work on the differential geometry of generalized space." It is dedicated to Dean Eisenhart and Professor Veblen. Indeed it follows very closely the content, notation and arrangement of the former's *Riemannian Geometry.* But it is purposely more elementary, less rigorous and less complete. Beside the basic essentials there are chapters on congruences and orthogonal ennuples, on hypersurfaces in euclidean and Riemannian space and on general subspaces. At no point does the book venture outside the domain of Riemannian geometry proper.

The following features are noteworthy:

(1) An introductory chapter giving a résumé of theorems of algebra and analysis used freely in the later chapters.
(2) A pleasantly restrained use of the notation of classical vector analysis as in the author's earlier *Differential Geometry of Three Dimensions.*
(3) Sections based on some original work by the author on quadric hypersurfaces.
(4) The very desirable use of R. Lagrange's generalized covariant differentiation in the treatment of subspaces.
(5) 125 exercises distributed among the chapter endings, some of which include important subject matter not in the text proper.
(6) An appended historical note condensed from an address (1932) by the author in which reference is made to the various generalizations of Riemannian geometry.
(7) A bibliography extending from the year 1827 with 132 entries and references to bibliographies by other authors.
(8) The excellent typography and general physical characteristics.

It will be a disappointment to some that the book was not constructed along more original and more stimulating lines although this is much to demand of an introductory text. Greater effort might have been made to lessen the emphasis on formalism which is so difficult to avoid in this field. Basic concepts could have been presented more carefully and given richer meaning. For example, increments and differentials are confused occasionally in accordance with a time-honored but deplorable custom; and the tensor concept as presented here is little more than a law of transformation. The notion of tangent space of differentials would be of assistance on both counts. Important existence theorems and elementary topological considerations would have been welcome. For example, the generalized Gauss-Codazzi equations are written down but their fundamental importance as existence conditions left unmentioned, and throughout the book the word "space" is never precisely defined. An obvious omission on the formal side is that of the alternating ϵ-tensors.

The book is neither for dilettantes nor advanced students of the subject. But to those who are seeking an introductory treatment of textbook character to serve perhaps as a sequel to the author's *Differential Geometry* it is to be recommended.

J. L. VANDERSLICE