Hardy and Littlewood had earlier derived the result that every sufficiently large odd number is the sum of three odd primes, but under the assumption of an unproved theorem concerning the distribution of zeros of a certain transcendental function. Since Schnirelmann, Winogradoff has proved the Hardy-Littlewood theorem without any unproved assumption, thus including both the Hardy-Littlewood theorem and Schnirelmann's result. This is the present status of Goldbach's conjecture that every even number greater than 4 is the sum of two odd primes.

The theorems of Chapters 3 and 4 are of a newer and less well known type than those of the first two chapters. An illustration is the following theorem:

THEOREM 96 (Khintchine-Erdös). *Let $\mathfrak{A}$ be a set of positive integers a, and let $\mathfrak{B}$ be a set of positive integers b enjoying the property that a certain positive integer l exists such that every positive integer is a sum of l or fewer numbers b. Let $\mathfrak{S}$ be the set of all numbers of the form a or* $a+b$, $A(x)$ *the number of a's* $\leqq x$, $S(x)$ *the number of s's* $\leqq x$, *and, for some* α, $(0<\alpha<1)$, $A(x)\geqq\alpha$ *for all x greater than zero. Then, for all* $x>0$, $S(x)\geqq\alpha(1+(1-\alpha)/2l)x$.

The proof of this theorem is extremely short and involves a minimum of mathematical prerequisites. It gives almost the impression of an exercise in formal logic.

By specializing the set $\mathfrak{B}$, many important results are obtained, such as:

For a given positive integer $a>1$ *and for a positive integer* $x>2$, *the number of solutions of* $p+i^a\leqq x$, *p a prime, for any integer* $i>0$, *is greater than* $x/c(a)$, *where* $c(a)$ *is a constant dependent only on a; and a similar theorem for* $p+a^i\leqq x$.

To another theorem by Khintchine, Theorem 110, which belongs to the same general range as Theorem 96, Landau pays the compliment: "Der Kintchinesche Beweis ist elementar und doch ein sehr kompliziertes grosses Kunstwerk."

This book is a fitting memorial to a mathematician who was at the same time a leader in research and unsurpassed in his ability and eagerness to make the most advanced work done by others available to a larger group.

A. J. KEMPNER

Zur Differentialgeometrie der Kurven und Flächen fester Breite. By Hans Bückner. (Schriften der Königsberger gelehrten Gesellschaft, vol. 14, no. 1.) Halle, Niemeyer, 1937. 22 pp.

This pamphlet gives a readable and very brief account of this subject, based on the support function of the curve or surface. New results include lower bounds for the area of a surface of constant breadth and for length of a space curve of constant breadth; characterization of a curve of constant breadth as an oval for which any two of the support normals intersect within the closed region bounded by the curve; an analogous result for surfaces; and an equation connecting lengths and areas of the two parts into which a support normal divides a curve of constant breadth and the included area.

E. H. CUTLER

The Theory and Construction of Nondifferentiable Functions. By A. N. Singh. (Lucknow University Studies, no. 1, Faculty of Science, Session 1934–1935.) Lucknow, Newul Kishore Press, 1935. 7+110 pp.

The author of this tract has published numerous papers on nondifferentiability and curves without tangents since 1924, mostly in Indian periodicals, though a couple of papers have found their way to this country. In the present volume, which is based upon four lectures delivered at Lucknow University, he gives a synopsis of the work done in this field. The first lecture contains a brief history of the earlier attempts to construct nondifferentiable functions leading up to the example of Weierstrass. He discusses the method of Dini and the various series definitions, including the recent one due to van der Waerden. In the second lecture he treats functions defined geometrically, in particular, the curves of Bolzano, von Koch, Peano, Hilbert, Kaufmann, and Besicovitch, whereas the general method of Knopp is only mentioned. The third lecture contains his own arithmetical definitions which attach in part to earlier work of Peano and E. H. Moore. The last lecture deals with various properties of the derived numbers of nondifferentiable functions. In closing, let me add that anybody who is giving a course in real variables will find this little tract useful.

EINAR HILLE

Moderna Teoria delle Funzioni di Variabile Reale. Part 2. *Sviluppi in Serie di Funzioni Ortogonali.* By G. Vitali and G. Sansone. (Monografie di Matematica Applicata per cura del Consiglio Nazionale delle Ricerche.) Bologna, Zanichelli, 1935. 6+310 pp.

This second volume is a very valuable continuation of the first part which was reviewed in vol. 43 (1936), p. 15, of this Bulletin. It is devoted to expansions in orthogonal series and takes up quadratically integrable functions, Fourier series, Legendre series, Laguerre and Hermite series, and the Stieltjes integral. The first chapter gives the primary notions on Hilbert space, orthogonality, linear independence, approximation, convergence in the mean, and expansions in orthogonal series including the closure theorem of Vitali which serves as the basis for the discussion in the special cases. This is followed by a discussion of Fourier series, including convergence in the mean, local convergence criteria, Fejér and Poisson summability, and the Fourier integral (Fourier transforms are just mentioned). The treatment of Legendre series starts with an adequate discussion of the basic properties of the Legendre polynomials through the asymptotic formulas and leads up to Hobson's convergence theorem. The fourth chapter, dealing with Laguerre and Hermite series, is perhaps the most valuable in the whole book since these series are normally not discussed in standard texts on analysis. It presents the fundamental properties of the polynomials, including their asymptotic behavior, and ends with the convergence theorems of Stone and of Uspensky for Hermite series. The last chapter, which has very little contact with the rest of the book, gives a discussion of the Stieltjes integral with applications to the theory of distribution functions and their characteristic functions. There is a large bibliography. The treatment is up-to-date, rigorous without being heavy, and the book can be strongly recommended to those who have to give courses in real variables or classical mathematical physics.

EINAR HILLE

Summable Series and Convergence Factors. By C. N. Moore. (American Mathematical Society Colloquium Publications, vol. 22.) New York, American Mathematical Society, 1938. 6+105 pp.

Fairly early in the development of the theory of summability of divergent series, the concept of convergence factors was recognized as of fundamental importance in the subject. One of the pioneers in this field was C. N. Moore, the author of the book under review. He first introduced the name "convergence factors" in this connection, published some of the first convergence factor theorems, and has been one of the chief investigators in the subject since then. It is therefore appropriate that the first