ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

277. Mr. G. R. Kraus and Professor J. H. Neelley: *A classification of rational plane cubic curves.*

This paper gives methods of distinguishing the types of the cubic curve with special reference to the degenerate types. Using parametric notation the authors have shown how the different degenerate types may be detected and hence how each may be expressed. (Received May 25, 1937.)

278. Dr. J. H. Curtiss: *A note on the Cesàro method of summation.*

The following results are obtained: (1) If the sequences $\{S_{\lambda n+K}\}$ are respectively summable (C, α) to the values s_K, $K=0, 1, \cdots, \lambda-1$, for some $\alpha>0$, then the sequence $\{S_n\}$ is summable (C, α) to the value $(s_0+s_1+\cdots+s_{\lambda-1})/\lambda$. (2) If the series $\sum \overline{U}_{\lambda n+K}$ are respectively summable (C, α) to the values u_K, $K=0, 1, \cdots, \lambda-1$, for some $\alpha>-1$, then the series $\sum \overline{U}_n$ is summable to the value $u_0+u_1+\cdots+u_{\lambda-1}$. Examples are given to show that the theorems cannot be strengthened. (Received May 6, 1937.)

279. Dr. W. E. Sewell: *The derivative of a polynomial on various curves of the complex plane.*

Let $P_n(z)$ be a polynomial of degree n in $z=x+iy=re^{i\theta}$ and let $|P_n(z)| \leqq M$ on C. If C is an epicycloid or hypocycloid expressed in the usual parametric form with a divisible by b, then $|P_n'(z) \sin [a\phi/(2b)]| \leqq Mn/(2b)$, z on C. If C is a cardioid with polar equation $r=a(1-\cos\theta)$, then $|P_n'(z) \sin \theta/2| \leqq Mn/a$, z on C. If C is a rose curve with polar equation $r=a\cos k\theta$, then $|P_n'(z)| \leqq M(n+1)/a$, z on C. Of course these evaluations are independent of the position of the curves in the plane. (Received May 21, 1937.)

280. Professor D. V. Widder: *The successive iterates of the Stieltjes kernel expressed in terms of the elementary functions.*

The Stieltjes kernel is the function $H_1(x, y)=(x+y)^{-1}$. Its successive iterates are defined successively by $H_n(x, y)=\int_0^\infty H_1(x, t)H_{n-1}(t, y)dt$. It is proved in this note that $H_n(x, y)$ can be expressed in terms of the elementary functions. In fact it is a linear combination of a finite number of the functions

$[\log x-\log y]^{2k}/[(2k)!(x+y)]$ and $[\log x-\log y]^{2k+1}/[(2k+1)!(x-y)]$ with $k=0, 1, 2, \cdots$. The constants of combination are the coefficients in the Maclaurin expansion of $(\pi z/\sin\pi z)^n$. (Received May 7, 1937.)

281. Mr. O. H. Hamilton: *Fixed points under homeomorphisms of continua which are not connected im kleinen.*

It is shown that if M is a compact continuum in a metric space and if M contains no indecomposable continuum and contains no continuum which is the sum of two continua whose common part is disconnected, then every reversible continuous transformation of M into a subset of itself leaves some point of M fixed. It follows that if N is a compact continuum in the plane which contains no indecomposable continuum, does not separate the plane, and contains no domain, then every reversible continuous transformation of M into a subset of itself leaves some point of N fixed. Furthermore, if D is a compact simply connected domain in the plane whose outer boundary contains no indecomposable continuum, then every reversible continuous transformation of $\bar{D}$ into itself leaves some point of $\bar{D}$ fixed. An example is given of a compact acyclic continuous curve in the plane which cannot be carried into itself by any continuous transformation except the identical transformation. (Received May 15, 1937.)

282. Mr. V. G. Iyer: *On effective sets of points in relation to integral functions.*

Let $f(z)$ be an integral function of finite order ρ and type $\kappa(f)$. A sequence $[z_n]$ of distinct points having infinity as its sole limit point is said to be an effective set or E-set for $f(z)$ if, as n becomes infinite, lim sup $\log|f(z_n)|/|z_n|^\rho =\kappa(f)$. In this paper, the main problem is concerned with the determination of a sequence $[z_n]$ which forms an E-set for *each function* $f(z)$ of the class $C(\rho, d)$ of all functions of order ρ and type *less* than d, ρ and d being two positive numbers. It is shown that the zeros of the function $\sigma_\rho(z)=\prod^\infty_{-1}(1-z^n/n^{\alpha n})$, where $\rho=2/\alpha$, form an E-set for the class $C(\rho, 1/\rho)$ from which an E-set can be derived for $C(\rho, d)$, d arbitrary, by means of an easy transformation. The idea involved in the paper was suggested by the fact that the lattice-points of the plane form an E-set for the class $C(2, \pi/2)$. (Received June 1, 1937.)

283. Professor N. H. McCoy: *Subrings of direct sums.*

In a joint paper with Deane Montgomery (abstract 43-3-135), it was recently shown that a commutative ring R, every element a of which satisfies the conditions $pa=0$, $a^p=a$, where p is a fixed prime, is isomorphic to a subring of a direct sum of $GF(p)$. The primary purpose of the present paper is to establish several theorems of this nature, at least one of them being a direct generalization of the theorem stated. (Received May 29, 1937.)

284. Miss Annita Tuller: *The measure of transitive geodesics on certain three-dimensional manifolds.*

It is the object of this paper to prove that almost all geodesics on certain three-dimensional manifolds are transitive. The metric $ds^2=(dx^2+dy^2+dz^2)/$

$(1-x^2-y^2-z^2)^2$ defines a hyperbolic geometry in the interior of the unit sphere S. The three-dimensional manifolds treated are those defined by considering as identical the points congruent to each other under certain groups of the rigid motions of this geometry, groups which are properly discontinuous in S but cease to be properly discontinuous on S. In particular, it is proved that almost all geodesics are transitive if the manifold is closed. This result is extended, however, to a more general case. Hopf (Mathematische Annalen, 1930, pp. 710–716) has shown that for certain dynamical systems almost all motions are either stable or unstable; stable in the sense that the motion returns arbitrarily close to any position formerly occupied, unstable in the sense that it stays outside any finite domain after a finite length of time. It is proved that if almost all geodesics are stable, then almost all of them are transitive. (Received June 2, 1937.)

285. Mr. W. H. Ingram: *On the numerical solution of integral equations*. Preliminary report.

Certain theorems on matrices due to Aitken, Duncan, and Collar are made the basis of a new method for the numerical solution of linear integral equations. Alternatively, the results can, in part, proceed from a result by Hilbert on the s-fold faltung of a bilinear form (Goettinger Nachrichten, 1906, p. 161). In the case of the Fredholm integral equation with symmetric kernel, the first characteristic function is given by the s-fold definite integral of $K(x, t_s)K(t_s, t_{s-1}) \cdots K(t_2, t_1)\xi(t_1)dt_1 \cdots dt_s$ where $\xi(t)$ is arbitrary and where s need not be greater than 4 for excellent accuracy in the case of the vibrating string, for instance. The corresponding characteristic number is given by the ratio of the s-fold to the $(s+1)$-fold such integral. Successive characteristic functions and numbers are obtained with kernels modified by the successive imposition of constraints eliminating the last mode. It would appear that the theory can be extended to non-symmetric kernels for which the characteristic numbers come in conjugate complex pairs. (Received June 5, 1937.)

286. Professor V. W. Adkisson: *Plane peanian continua with unique maps on the sphere and in the plane.*

The plane peanian continuum M is said to have a unique map on a spherical surface (or a plane) if and only if for any topological image M' of M on a sphere (or plane) S' and any topological image M'' of M on a sphere (or plane) S'' every homeomorphism of M' into M'' can be extended to a homeomorphism of S' into S''. The following theorems are proved. Theorem 1. The plane peanian continuum M has a unique map on a sphere if and only if one of the following conditions holds: (1) M consists of either a simple arc or triod, (2) M contains one cyclic element C which is a maximal triply connected cyclic curve of M, and $M-C$ consists of at most a countable number of arcs, $a_1, a_2, \cdots$, such that $\bar{a}_i \cdot \bar{a}_j = 0 (i \neq j)$ and each $\bar{a}_i \cdot C$ is a single point which lies on only one bounding circuit of C, provided that if C is a simple closed curve then $M-C$ is at most a simple arc. Theorem 2. The plane peanian continuum M has a unique map in the plane if and only if M is a simple arc, a triod, a simple closed curve, or if M contains a closed 2-cell C and $M-C$ consists of at most a countable number of arcs $a_1, a_2, \cdots$, such that $\bar{a}_i \cdot \bar{a}_j = 0 (i \neq j)$ and each $\bar{a}_i \cdot C$ is a single point which lies on the only bounding circuit of C. (Received July 1, 1937.)

287. Professor A. A. Albert: *p-algebras over a field generated by one indeterminate.*

Let F be any perfect field of characteristic p, x be an indeterminate over F, K be algebraic over F. In particular the most interesting case is that where F is any finite field. It is shown that K is separable over $F(y)$ where y is the indeterminate whose p th power is x. The author considers division algebras of degree p^n over a centrum K as above and proves them all cyclic and with exponent equal to degree. Moreover all such algebras have a common splitting field $K(u)$, $u^{p^n} = y$. This paper will appear in full in the October number of the Bulletin. (Received July 2, 1937.)

288. Dr. Reinhold Baer: *Abelian fields and duality of abelian groups.*

E. Witt has recently characterized the finite abelian extensions of commutative fields by means of invariants. Witt's theory may be extended, almost without modification, to all those extensions whose group is abelian (finite or not). This may be done by the simple device of substituting Pontrjagin's duality theory for the classical theory of characters of finite abelian groups. (Received July 9, 1937.)

289. Dr. R. P. Boas: *Asymptotic relations for derivatives.*

One case of a well known theorem of Hardy and Littlewood states that if $f(x) \epsilon C^n(0, \infty)$, and if, as $x \to \infty$, $f(x) = o(x^r)$ and $x^n f^{(n)}(x) < O(x^r)$, $(r \geqq 0)$, then $x^k f^{(k)}(x) = o(x^r)$, $(1 \leqq k \leqq n-1)$. This is generalized, both by replacing $x^n f^{(n)}(x)$ by a more general linear differential form, and by replacing x^r by $\phi(x)$, a function of suitably regular behavior. (Results of the second kind overlap, but are not included in, more general results of Hardy and Littlewood.) In a later paper by D. V. Widder and the author, applications to the theory of the iterated Stieltjes transform will be given. (For this transform, see D. V. Widder, Proceedings of the National Academy of Sciences, vol. 23 (1937), pp. 242–244.) (Received July 7, 1937.)

290. Professor S. S. Cairns: *Normal coordinates for extremals transversal to a manifold.*

Consider a positive definite regular calculus of variations problem, $\int F(x, \dot{x})dt$, where $F(x, r)$ is defined on an n-manifold R for $(x) = (x^1, \cdots, x^n)$ neighboring a point (x_0) and for $(r) = (r^1, \cdots, r^n) \neq 0$. In terms of *normal coordinates* with origin (x_0), the extremals having (x_0) for initial point are represented, in a certain neighborhood, by linear equations $y^i = c_i s$, $(0 \leqq s < s_0)$, where s is the arc length and the c_i are direction cosines. In this paper normal coordinates are obtained under weaker hypotheses than heretofore used. The author defines *normal coordinates (z) with respect to an m-manifold*, $(0 < m < n)$, M, on R. In terms of (z), M is defined, in a certain neighborhood, by $z^{m+1} = \cdots = z^n = 0$, and the general extremal cut transversally by M at its initial point is defined by $z^i = a_i$ $(i=1, \cdots, m)$, $z^j = c_j s$, $(j=m+1, \cdots, n)$, $(0 \leqq s < s_0)$; wherein (a, c) are constants for each extremal, $c_j c_j = 1$, and s is the arc length from M. As the c's vary over their $(n-m-1)$-sphere, the a's being fixed, one obtains the transversal extremals with a common initial point. They cover an $(n-m)$-manifold, differentiable save, in general, at the initial point. Necessary and sufficient conditions on $F(x, r)$ that this $(n-m)$-manifold be differentiable without exception for every m-manifold M are obtained. (Received June 1, 1937.)

291. Professor Leonard Carlitz: *An analog of the von Staudt-Clausen theorem.*

Put $t/\psi(t) = \sum B_m t^m / g(m)$, where $\psi(t)$ is the function defined in the Duke Mathematical Journal, vol. 1 (1935), p. 137, and $g(m)$ is a certain polynomial. The coefficients B_m have properties analogous to those of the ordinary Bernoulli numbers; some of these properties were discussed in the paper referred to. In the present paper some arithmetic properties of B_m are developed. The main result is that $B_m = G_m - e\sum 1/P$, where G_m is a polynomial, e an integer, and the summation is over a certain set of irreducible polynomials. The method of proof depends on certain ideas due to A. Hurwitz (Mathematische Annalen, vol. 51 (1899), pp. 196–226). (Received July 8, 1937.)

292. Professor R. V. Churchill: *The solution of linear boundary value problems in physics by means of the Laplace transformation.* II.

As an application of the theory in Part I (to be published in the Mathematische Annalen) the temperature distribution function is determined for a composite slab of finite thickness with zero initial temperature and variable surface temperature. The function is expressed in different real integral forms as well as in the form of a series, and conditions on the variable surface temperature are established under which these forms represent the solution of the boundary problem. The method of solution gives the Laplace transform of the temperature function, and by applying known Tauberian theorems to this transform some interesting properties of temperatures and heat transfer in composite slabs and bars are shown. (Received July 8, 1937.)

293. Professor H. S. M. Coxeter: *A graphical representation for the simple group of order* 504.

This paper concerns a group generated by three operators whose products in pairs are involutorial, as is also the product of all three (in a definite cyclic order). If m, n, p, are the periods of the three generators, the group is denoted by $G^{m,n,p}$. In particular, $G^{3,5,5}$, $G^{3,7,9}$, $G^{5,5,5}$, $G^{3,7,15}$ are the simple groups of orders 60, 504, 660, 12180, respectively. By regarding $G^{m,n,p}$ as a factor group of the group generated by inversions in three circles which form a curvilinear triangle of angles $\pi/2$, π/m, π/n, the author obtains a representation on a network of such triangles, in the manner that was first described by Dyck. By drawing new circles through the vertices where right angles occur, one derives a "semi-regular map" of m-gons and n-gons, in which the third generator of

$G^{m,n,p}$ appears as a "glide" along one of these circles. When the order of the group exceeds 500, the drawing is usually too extensive to be practicable; but the map for $G^{3,7,9}$ is particularly elegant. It consists of 84 triangles and 36 heptagons, and forms an unorientable surface of connectivity 8. (Received June 29, 1937.)

294. Dr. Aaron Fialkow (National Research Fellow): *The Riemannian curvature of a hypersurface.*

The author proves the following generalization of a well known theorem of Riemannian geometry. When none of the lines of curvature at a point of a V_n in a V_{n+1} are tangent to a null vector, the difference of the Riemannian curvatures of the V_n and V_{n+1} for the orientation determined by the directions of two lines of curvature at the point is numerically equal to the product of the corresponding normal curvatures; the sign is determined by the character of the normal to V_n in V_{n+1}. By means of this result, a characteristic property of flat spaces is derived. (Received July 8, 1937.)

295. Professor Tomlinson Fort: *The calculus of variations applied to Nörlund's sum.*

The principal solution of the linear difference equation $\Delta y(x) = f(x)$ which Nörlund calls a "sum" has many resemblances to an integral. The purpose of the present paper is to apply methods of the calculus of variations to the problem of minimizing such "sums" taken between constant limits. An analog to Euler's equation is obtained. There is also a treatment of the second variation with the development of further necessary conditions. (Received June 17, 1937.)

296. Dr. Bernard Friedman: *Analyticity of equilibrium figures of rotation.*

The possible forms of relative equilibrium of a homogeneous gravitating mass of liquid, when rotating about a fixed axis with constant angular velocity, are given by an integral equation (to be specified later) in which the domain of integration depends upon the equilibrium figure. Let the z-axis be the axis of rotation, ω the angular velocity, and R the region containing the rotating mass. Then at equilibrium the following equation holds: $(\omega^2/2)(x^2+y^2) - \int_R dV/\overline{PQ}$ $=$ constant, where $P(x, y, z)$ is any point on the surface of R, and Q, the integration point, runs through R. Lichtenstein proved that, at all points where the apparent gravity is not zero, the surface possesses continuous derivatives of all orders. In this paper, it is proved that the surface is also analytic. The method is essentially that used by E. Hopf in proving the analyticity of solutions of elliptic differential equations of the second order. (Received June 22, 1937.)

297. Professor Edward Kasner: *The geometry of conformal symmetry (Schwarzian reflection).*

A general analytic curve C defines a symmetry transformation S which is a conformal covariant. Let C_1 be any curve going through a point P of C and let C_2 be the image of C_1 in C. In this paper fundamental theorems are obtained stating relations between the curvatures γ, γ_1, γ_2 and their successive derivatives with respect to arc. By definition, C is the conformal bisector of the curvilinear angle C_1C_2. In particular if C_1C_2 is a horn angle, the quantities γ, $d\gamma/ds$, $d^2\gamma/ds^2$ assume mean values, but not the higher derivatives. (In the equilong theory *all* derivatives take mean values.) To be published in Annals of Mathematics, October, 1937. (Received July 3, 1937.)

298. Professor Edward Kasner: *The two conformal invariants of fifth order.*

Associated with a first order contact of two curves (horn angle H_2), there is a unique absolute conformal invariant of third order, involving curvatures and their first derivatives. (See Proceedings of the National Academy of Sciences, April, 1936.) If the contact is of second order (horn angle H_3), the unique conformal invariant is of fifth order. If two curves are orthogonal (general curvilinear right angle) an invariant, also of fifth order, results. The two invariants may be interrelated by the theory of symmetry (Schwarzian reflection). When one of the curves in H_3 is assumed to be a circle, one obtains Mullins' inversive invariant of a general curve. Relative invariants are also studied. (Received July 3, 1937.)

299. Professor D. H. Lehmer: *An application of Schläfli's modular equation to a conjecture of Ramanujan.*

The conjecture in question may be quoted as follows. Let $q=5$, 7, or 11. If n is chosen so that $24n-1$ is divisible by q^a, then the number $p(n)$ of unrestricted partitions of n is divisible by q^a. In the present paper the author shows how to use to advantage the so-called Schläfli's elliptic modular equation connecting $f(mz)$ with $f(z)$ (where $f((-D)^{1/2})$ is either of the two fundamental class-invariants associated with D), to the problem of verifying the above conjecture by means of the Hardy-Ramanujan series for $p(n)$ whose application for every n has been justified recently by Rademacher. The method is applied to four new cases in which it is proved that the numbers $p(n)$ for $n=1224$, 2052, 2474, and 14031 are respectively divisible by 5^4, 11^3, 5^5, and 11^4. All four results are in accord with the conjecture of Ramanujan. (Received June 28, 1937.)

300. Mr. Ingo Maddaus: *On completely continuous linear transformations.* Preliminary report.

Let E be a Banach space with the additional property that every element g is of the form $\int_0^1 f(t)d_t\lambda(g;t)$, where $f(t)$ is a continuum of elements of the space, each of unit norm, and $\lambda(g;t)$ is a bounded linear operation in g for each t. Then any completely continuous linear transformation on a Banach space to a space of type E is the strong limit of a sequence of linear transformations each of which transforms its domain into a space of finite dimension. (Received July 9, 1937.)

301. Mr. Philip Newman: *The maximum of a superharmonic function.*

A superharmonic function is known to take its minimum on the boundary.

By means of "harmonic improvement" and by forming the minimum function of a finite set of superharmonic functions, a sequence of functions can be constructed converging uniformly to a superharmonic function which takes its maximum on the boundary and has the same boundary values as the given superharmonic function. Employing Perron's boundary functions (Mathematische Zeitschrift, vol. 18 (1923), pp. 42–54) the foregoing theorem can be used to demonstrate that the upper and lower functions for a boundary value problem have a common limit function which is at once seen to be continuous and harmonic. (Received July 8, 1937.)

302. Professor Gordon Pall: *Applications of generalized quaternions to regularity of ternary quadratic forms.* Preliminary report.

Let $\lambda=1$, 2, or 3. Let n be a positive integer, $n\equiv 1 \pmod 8$ if $\lambda=1$ or 2, $n\equiv 1 \pmod{24}$ if $\lambda=3$. In the equation $n=\lambda x_1^2+x_2^2+x_3^2$ all the solutions in integers x_h satisfy either A or B, where A implies x_2 or $x_3\equiv 0 \pmod{4\lambda}$ for $\lambda=1$, 2 and $0 \pmod{2\lambda}$ for $\lambda=3$; B implies x_2 or $x_3\equiv 2\lambda \pmod{4\lambda}$ for $\lambda=1$, 2, and $\equiv\lambda \pmod{2\lambda}$ for $\lambda=3$. If n is a square m^2, $m>0$, such that $(-\lambda|m)=1$, then all proper solutions (at least) are of type A; while if $(-\lambda|m)=-1$, all proper solutions are of type B. But if n is not a square, there are equally many solutions of each type A and B. The proof is based on properties of $\bar{t}at$, where a and t are quaternions in which $i^2=-1, j^2=-\lambda, \cdots$; and there are immediate applications to the proof that certain forms (for example, $x^2+3y^2+36z^2$) are regular though they belong to genera of more than one class. (Received June 15, 1937.)

303. Professor Gordon Pall: *The quaternion congruence $\bar{t}at\equiv b$ (mod g) and the equation $h(8n+1)=x^2+y^2+z^2$.*

If h contains no factor >1 of the form $8f+1$, then every solution of $h(8n+1)=x_1^2+x_2^2+x_3^2$ in coprime integers x_m belongs to one of two classes of residues $(x_1, x_2, x_3) \pmod{4h}$. If $8n+1$ is a square s^2, $s>0$, then all the solutions are in one class or the other according as $s\equiv 1$ or $3 \pmod 4$. But if $8n+1$ is not square, there are equally many solutions in each class. The proof depends on a study of the form $\bar{t}at \pmod g$ where a is a pure integral quaternion, t is an arbitrary integral quaternion, and g is a rational integer. If g is an odd prime dividing Na but not a, $\bar{t}at$ represents by choice of t exactly half the residues, besides $(0, 0, 0)$, of norm$\equiv 0 \pmod g$; unless $g\equiv 1 \pmod 8$ the other half are represented by $\bar{t}a't$, where a' is obtained from a by permuting or changing signs of i, j, k. Similar results hold if g is composite. If b is a pure quaternion, and $Nb\not\equiv 0 \pmod 4$, all solutions t of $\bar{t}at\equiv b \pmod 8$ satisfy one and only one of $\bar{t}t\equiv 1$ or $3 \pmod 4$. Conditions for solvability with $g=2^n$ or p^n are obtained. (Received June 15, 1937.)

304. Professor W. V. Parker: *On symmetric determinants.*

In this paper the following results are obtained. If D is a symmetric determinant of order $n>4$ with real elements, in which all principal minors of some order $k>3$ and also all principal minors of order $k-3$ are zero, then D is of rank $k-1$ or less. If $n>5$ and M is any principal minor of D of order $n-1$ and if the principal minors of D of orders k and $k-3$ are divided into two sets, first those which are minors of M and second those which are not minors of M; then if all minors in either set are zero, D is zero. (Received July 6, 1937.)

305. Professor W. V. Parker: *The characteristic roots of a matrix.*

If λ is a characteristic root of the matrix $A=(a_{ij})$ and if $R_i=\sum_{j=1}^n|a_{ij}|$, $T_j=\sum_{i=1}^n|a_{ij}|$, and $2S_i=R_i+T_i$, it is shown that the absolute value of λ does not exceed the greatest of the S_i. In case the matrix A is Hermitian, an upper limit for the characteristic root of least absolute value and a lower limit for the characteristic root of greatest absolute value are given also. (Received June 25, 1937.)

306. Dr. Moses Richardson: *On periodic transformations of complexes.*

Let K be an abstract simplicial complex and T a single-valued simplicial mapping of K on itself of period m which preserves incidence and dimension, where m is arbitrary. Let k be the complex obtained by identifying all elements of K which are congruent under the powers of T; let K^* be the subcomplex of K consisting of all invariant simplexes; and let k^* be the counterpart of K^* on k. Under certain combinatorial restrictions, the author determines the mod m Betti numbers of (a) k; (b) k mod k^*; and (c) K^*. For the case $m=2$, (a) and (b) have been discussed previously by P. A. Smith and the author. The method used is an extension of that of Smith. (Received July 3, 1937.)

307. Dr. J. B. Rosser: *The nth prime is greater than n* log *n.*

Counting 2 as the first prime, denote the nth prime by $p(n)$. For large n, $p(n)=n\log n+n\log\log n-n+O[(n\log\log n)/\log n]$, so that for large n, $p(n)>n\log n$. In this paper it is shown that for all positive n, $p(n)>n\log n$. As a consequence of this, one can prove that if B_n, C_n, and D_n are defined respectively for $n\geqq 2$ by $\sum_{m=1}^n 1/p(m)=\log\log n+B_n$, $\prod_{m=1}^n(1-1/p(m))=C_n/\log n$, and $\prod_{m=2}^n(1-2/p(m))=D_n/\log^2 n$, then $B_n>B_{n+1}$, $C_{n+1}>C_n$, and $D_{n+1}>D_n$. It is readily shown that all three sequences approach limits, and the numerical values of these limits are given to ten decimal places. These results will be used in the author's paper, *An improvement of Brun's method in number theory.* For completeness, the following theorems are also proved. If $n>3$, $p(n)<n\log n+2n\log\log n$. If $n>1$, $n\log n+n\log\log n-n-9n<p(n)<n\log n+n\log\log n-n+9n$. (Received June 26, 1937.)

308. Dr. A. C. Schaeffer: *Existence theorem for the flow of an ideal incompressible fluid in two dimensions.*

Let initial velocity components be defined throughout a plane region at a given initial time such that the divergence is zero, the normal component of the velocity at the boundary is zero, and certain smoothness conditions are satisfied. It is shown that a flow exists for all time which satisfies at the initial time the given initial conditions. The method of successive approximations is used, which allows one to construct the flow for all time. (Received June 21, 1937.)

309. Dr. A. R. Schweitzer: *On a classification of metrical relations in the foundations of geometry.*

This paper is in continuation of two preceding papers reported in this Bulletin, abstracts 43-1-53 and 43-1-92. The basis of the author's classification of metrical relations is the analogy between (1) order and metrical relations, (2) metrical relations on different levels of measurement. In the first category the author places the metrical analogs $E^{(S)}$, $E^{(B)}$, $E^{(I)}$, $E^{(K)}$ of his relations S, B, I, K. The relation E expresses types of equidistance of points from other points; in particular, the relation $E^{(K)}$ expresses congruence between two n-dimensional $(n+1)$-ads of points ($n=1, 2, 3, \cdots$). This analogy leads to an abstract empty bracket notation in which the relations S, B, I, K are replaced by empty brackets of the type "( )" which may be filled by $E^{(S)}$ or S, $E^{(B)}$ or $B, \cdots$, respectively. In the second category the author places as analogs of his relations, $\alpha_1\alpha_2 \cdots \alpha_{n+1}=k\cdot\beta_1\beta_2\cdots\beta_{n+1}$ and $\alpha_{jn}=k\cdot\beta_{jn}$, the relations, $SP(\alpha_1\alpha_2\cdots\alpha_{n+1})=k\cdot SP(\beta_1\beta_2\cdots\beta_{n+1})$ and $SP(\alpha_{jn})=k\cdot SP(\beta_{jn})$ where SP denotes the oriented metrical linear, areal, $\cdots$ "span," and the n-dimensional angular "span," respectively. The coefficient k denotes a real number, and in the latter type of relations the $(n+1)$-adic terms need not be cospatial. For $n=1$ reference is made to Pieri and Minkowski. (Received June 30, 1937.)

310. Dr. A. R. Schweitzer: *A theory of congruence in the foundations of geometry.*

Supplementing the author's n-dimensional ($n=1, 2, 3, \cdots$) oriented descriptive "K" systems outlined in previous reports in this Bulletin, sets of congruence axioms are constructed (1) primarily without, (2) with, reference to orientation. These systems are phrased in terms of undefined $(n+1)$-adic relations $E^{(K)}, K$ and $E_K^{(K)}, K$ respectively. This theory is shown to be equivalent to the well known congruence theory of Hilbert. In a second part of the paper a study is made of the analogy between the relations K and E based on the fact that both relations are effective between $(n+1)$-ads of points and both are formally of the nature of equality. This analogy leads to the construction of a general abstract theory $\mathcal{G}[K, (\ )]$, in terms of an empty bracket relation, which remains valid if the bracket is filled by K throughout or by E throughout. A congruence theory equivalent to the preceding theory is then obtained from $\mathcal{G}[K, (E)]$ by adding certain axioms in terms of K and E. In the dyadic case, only one (existential) axiom is required. This development represents on an absolute, axiomatic basis, the gradual transition from order to congruence in the foundations of geometry. (Received June 30, 1937.)

311. Mr. C. E. Springer: *On the metric differential geometry of a general surface in a linear space of four dimensions.*

This paper employs the tensor notation to extend the results of V. G. Grove (Transactions of this Society, vol. 39 (1936), pp. 60–70) to a general surface in S_4, using the differential equations of the surface in the form developed by Eisenhart (*Riemannian Geometry*, 1926, p. 189). Further, a study is made of the congruence which is determined by the line of intersection of the normal plane to the surface at a point and the plane determined by the two lines of intersection of planes of certain degenerate quadrics. The condition that this congruence be conjugate to the parametric net on the surface is obtained. Two geometrically defined reciprocal cubic cones are studied at a point on the surface. The surfaces for which these cubic cones coincide at every point have the property that the congruence is conjugate to them. (Received July 9, 1937.)

312. Mr. J. V. Wehausen: *Bounded sets in topological linear spaces.*

This paper is concerned with bounded sets in a topological linear space satisfying the Axioms 1, 3, 4, 5, 6 of John von Neumann (Transactions of this Society, vol. 37 (1935), p. 4). In such a space a linear operation which is continuous takes bounded sets into bounded sets; the converse is proved provided there exists a bounded open set in the domain, this latter condition, however, being sufficient for the existence of an equivalent (F)-metric. If the space is also convex (von Neumann, loc. cit., Axiom 7), this condition is both necessary and sufficient for the existence of an equivalent norm metric. By defining a weak topology in a norm vector space by the neighborhoods $U(F_1, \cdots, F_n; \epsilon) = E_x[|F_i(x)| < \epsilon]$, F_i linear functionals on the space, it is proved that no neighborhood is bounded, the topology is not metrizable, and the space is of the first category unless there exists a finite total set of functionals (Banach, *Opérations Linéaires*, p. 42) for the space, this being equivalent to finite dimensionality. (Received July 8, 1937.)

313. Professor F. H. Safford: *Magic squares of the fifth order.*

This paper presents the computation of all squares which can be formed from the first twenty-five natural numbers where all rows and columns have the same sum and all pairs of numbers symmetrically located as to the center have another constant sum, thus superseding the usual diagonal condition. The number of these squares is 3004, all independent and transformable in 128 ways each. The squares are listed with respect to any two independent and noncentral rows or columns, and the compilation has been deposited with the Department of Mathematics of the University of Pennsylvania. (Received July 8, 1937.)