Researches on Waring's Problem. By L. E. Dickson. Carnegie Institution of Washington, Publication No. 464, July, 1935. v+257 pp.

Waring's problem is now entering its last phase: to find U such that *every* positive integer is a sum of $U=U(k)$ integral kth powers $\geqq 0$. As has long been known $U(3)=9$. But it is also known that *some* integers demand exactly 9 cubes. In this sense 9 is the true universal U for $k=3$. The Hardy-Littlewood analysis gives a number $N=N(k)$, depending only on k, such that every *sufficiently large* positive integer is a sum of at most N kth powers; for example, $N(4)=21$, whereas Waring conjectured $U(4)=19$.

To dispose finally of Waring's conjecture it remains to determine U(k), universal in the sense that $U(3)=9$ is, and to indicate how the integers which require fewer than U(k) kth powers can be calculated. This goal is in sight, owing to recent advances due to Winogradov and R. D. James (using in part the results of Winogradov). When all this is done the curious may wish to know the number of representations as a sum of a prescribed number of kth powers, $k>2$, in continuation of the classic results for $k=2$. Nothing of any consequence has been accomplished in this direction.

Dickson is led to consider $I=I(k)=2^k+g-2$, where g is the greatest integer $<(3/2)^k$. "No exception," he states, "has been found to the empirical result that every positive integer is the sum of I kth powers, so that I is the ideal value for a universal theorem." It is easily seen that 2^kg-1 demands I kth powers. Reasons are given for stopping with U(17). For $k=7$ the Hardy-Littlewood N(k) is 192, while $I(k)=143$, and as k increases, $N(k)-I(k)$ increases rapidly, so that there is still plenty to shoot at: thus $N(13)=22639$, $I(13)=8384$. By an exhaustive analysis, Dickson finds $U(5)=54$, $U(7)=233$. For $k\geqq 11$ the new U(k) are less than the Hardy-Littlewood N(k). Thus after $k=10$ the new *universal* theorems are an improvement on the older *asymptotic* results—surely a striking advance.

The bulk of the book (pages 84–257) is a table giving the minimum decompositions of all integers from 3,470,000 to 3,600,000 into a sum of (positive) fifth powers of integers. Letters a, · · · , B are used to denote the fifth powers of 2, · · · , 20, making possible the printing of the table in four columns to the page. Thus, under the heading 3470 we find opposite 19 the entry 3dfhjmB, which states that 347019 is the sum of the fifth powers of 3 and the numbers denoted by d, f, h, j, m, B. To eliminate typographical errors (and possibly also to keep down expense), the entire book, with the exception of the preface, has been reproduced without typesetting or linotype from the original typescript.

E. T. Bell

Synthetische Geometrie. By H. Liebmann. Teubner, Leipzig, 1934. viii+119 pp.

This is volume 40 of Teubner's Mathematische Leitfäden written by Professor Liebmann of the University of Heidelberg. It is an elementary treatise on projective geometry, based on strictly axiomatic foundations implied by modern research and criticism, and is a valuable aid to students and teachers of this branch of mathematics.

Arnold Emch