Topologie. I. By C. Kuratowski. Warszawa-Lwow, Monografje Matematyczne, 1933. ix+285 pages.

This is the first volume of what the reviewer judges will be a three-volume treatise covering the whole field of topology. Vast though this undertaking may be, its success is completely assured by the amazingly masterful way in which the foundations are laid in this first volume. The object clearly is the presentation of all the major conclusions of topology in their proper logical sequence and at the same time in their simplest form and greatest generality. The degree of success attained by the author in this direction is indeed the most remarkable feature of the book; and after reading this volume, little doubt can remain but that when the project is completed it will find a central place among those works of mathematical literature which are of constant and lasting value.

The treatment is based on the undefined notion of the closure of a set (that is, the operation $\overline{A}$ of taking the set A together with all its points of accumulation) and on a system of axioms stated in terms of this notion. The organization of the material is such that the space becomes progressively more and more specialized as new axioms are added to the system. The first chapter deals with fundamental notions in what might be called general topological space, the second with separable metric spaces, and the third with complete spaces. The same plan is to be continued in the succeeding volumes where compact spaces, connected spaces, etc., will be treated. The methods of proof used in this volume are purely set-theoretic as distinguished from the combinatorial methods, which are not employed. The notation and symbolism used is so efficient that all proofs may be given in small space and they take on a decided "analytical" appearance. The brevity of the proofs, however, is more properly attributed to the ingenuity of the author and his excellent logical organization of the material.

After a brief introduction dealing with the fundamental operations of logic and of the theory of sets and with the notions of the cartesian product of sets and of functions or transformations defined on sets, the formal treatment begins in Chapter 1 with three axioms as follows: (I) $\overline{X+Y}=\overline{X}+\overline{Y}$, (II) if X is vacuous or a single point, $\overline{X}=X$, (III) $\overline{\overline{X}}=\overline{X}$. Upon this basis the author treats closed and open sets, boundary points and interior points, and neighborhoods of a point and localization of properties, dense sets, boundary sets, non-dense sets, sets of the first category, the property of Baire, development of sets in alternating series of closed sets, continuity of functions, and homeomorphisms. The author points out incidentally that these three axioms are equivalent to the first three axioms of Hausdorff.

Chapter 2, on separable metric spaces, is divided into five parts. The first of these is devoted to the introduction of the metric into the space, and in the course of the development two new axioms are added, axiom IV being an axiom of separation (normality) and axiom V an axiom giving a base for the space (perfect separability). Part B deals with problems concerning the power of sets. In part C there is presented practically the whole of the theory of dimension; and the fact that all of the principal results on this subject are here developed and proved in their full generality in less than 20 pages attests both the remarkable refinements which have been made in this theory within the past five years and also the perfect logical arrangement of the material by the author. The fourth part deals with cartesian products and sequences of sets and the final part considers in great detail the Borel sets and functions measurable B in their most general form.

The third chapter, which has to do with complete spaces, treats questions relative to sequences of sets, extensions of functions, projective sets, analytic sets, totally imperfect sets, etc.

The value of the present volume lies not only in its usefulness as an introduction to the fundamentals of topology but more notably in the completeness with which the topics treated have been handled. Even the latest results are to be found here in what appears to be exactly their proper setting relative to the rest of the subject, and full advantage has been taken of even the most recently developed methods in refining the treatment.

G. T. WHYBURN

Symbiose, Parasitisme et Évolution. By V. A. Kostitzin. (Actualités Scientifiques et Industrielles, 96.) Paris, Hermann, 1934. 40 pp.

It is only very recently that the mutual influences of various species of plants and animals have been studied mathematically. Volterra's *Théorie Mathématique de la Lutte pour la Vie* (1931) was, of course, the pioneer treatise on the subject; and, aside from sections of Lotka's *Elements of Physical Biology* (1925), further discussion has been almost completely confined to periodicals.

Kostitzin's stimulating booklet is valuable, less because of new contributions to the mathematical methods than because of his enlargement of the scope of such studies,—particularly by a new emphasis on symbiosis (including commensalism). Even here—so complicated does the mathematics become for problems closely approximating to nature—the most difficult case considered is that of two species, each divided into two age-groups, with rates of birth and death dependent both on age and on the extent of the mutual aid. Kostitzin inquiries into periodic and stationary solutions of the differential equations, and into the stability of the latter type.

Equal emphasis is given to parasitism (harmful symbiosis). One section tells of the aid which mathematical analysis gave in determining how chlorogaster, a parasite of pagurians, develops in the hosts.

The treatment,in a booklet of this size, must too often be merely suggestive. Thus, we should be grateful for a thorough discussion of the remark that approximate agreement of periods usually signifies instability of a mechanical system, stability of a biological one.

A recent note by the author in the Paris Comptes Rendus (vol. 195, p. 1219) gives a suggestion of the wide applicability of this type of biomathematics. Therein he makes conjectures on the symbiotic periodicity of plant, animal, and inorganic matter of the earth as a whole; and on that of sediment, animal life, and H_2S in the Black Sea.

As this book shows, the way is open for a very salutary symbiosis (at the Christmas season we may prefer the term "commensalism") of mathematics and biology.

E. S. ALLEN