SHORTER NOTICES

Tafeln für die Differenzenrechnung sowie für die Hyperbel-, Besselschen, Elliptischen, und anderen Funktionen. By Keiichi Hayashi. Berlin, Springer, 1933. 66 pp.

This book contains ten major tables and several minor ones. The first and most extensive of the major tables bears the title *Tafeln für die Differenzenrechnung.*

By starting with the Taylor expansion of $f(x\pm nh)$ and using the definition of differences, it is an easy matter to derive formulas for differences in terms of derivatives. The most general formula of this kind is

$$\Delta^m_{x\pm nh} = \frac{[\cdots]h^m}{m!} f^{(m)}(x) \pm \frac{[\cdots]h^{m+1}}{(m+1)!} f^{(m+1)}(x) + - \cdots$$

where the brackets represent coefficients which depend upon m and n. For example,

$$\Delta^5_{x-2h} = \frac{[120]}{5!} h^5 f^{v}(x) - \frac{[-360]}{6!} h^6 f^{vi}(x) + \frac{[1680]}{7!} h^7 f^{vii}(x) - + \cdots$$

$$= h^5 f^{v}(x) + \tfrac{1}{2} h^6 f^{vi}(x) + \tfrac{1}{3} h^7 f^{vii}(x) + \cdots .$$

The main table under the calculus of differences contains the numerical values of the bracket coefficients for $m=1$ to $m=12$ and $n=0$ to $n=50$. A second table gives the values of the coefficients

$$\frac{[\cdots]}{m!},\ \frac{[\cdots]}{(m+1)!},\ \cdots;$$

and a third table gives the values of

$$\frac{[\cdots]}{m!} h^m,\ \frac{[\cdots]h^{m+1}}{(m+1)!},\ \cdots$$

for $h=0.01$.

The second major table is entitled *Interpolationstafeln für* $J_0(x)$, $J_1(x)$. The main table gives the values of certain coefficients used in computing differences from the values of Bessel functions. Other tables give the values of $J_0(nh)$, $J_1(nh)$, $\cdots$, $J_6(nh)$ and of $J_n'(0)$, $J_n''(0)$, $\cdots$, $J_n^{xiv}(0)$.

Other major tables include tables of $e^{\pi x}$, $e^{-\pi x}$, $\sinh \pi x$, $\cosh \pi x$ by steps of 0.01 from $x=0$ to $x=10$; ten-place tables of $Y_0(x)$, $Y_1(x)$ by steps of 0.01 from $x=16$ to $x=25.50$; the numerical values of $Y_0'(x)$, $Y_0''(x)$, $\cdots$, $Y_0^{xiv}(x)$ for $x=16$ to $x=25$; the values of x^4, x^5 by steps of 1 from $x=1$ to $x=1000$; the values of x^{11}, x^{12} from $x=1$ to $x=100$; and ten-place tables of E, E' by steps of 0.001 for k^2. A few other smaller tables are included in the book.

The book is beautifully printed on paper of excellent quality. It should be of considerable value to all who work with Bessel functions.

J. B. Scarborough