tary forms follow immediately from the method we have used throughout this paper.

10. *Bertini-Bertini.* Given $I_\alpha : O_1^6 \cdots O_8^6$ and I_β as in §4. The order of the transformation is 34, and by repetition of what has been done above we can discuss this transformation completely.

If fundamental elements of one or both Cremona plane involutions lie on the line $c \equiv \alpha\beta$ the preceding results must be modified. The details are not difficult in each particular case, but the large number of possible forms cannot be considered here.

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A SEPARATION THEOREM*

BY W. A. WILSON

Various writers on topology have had occasion in the course of their work to prove lemmas of the following general nature. Given sets A and B lying in a connected space Z, it is possible to express Z as the union of two continua M and N such that $N \cdot (A - A \cdot B) = M \cdot (B - A \cdot B) = 0$, provided that A, B, and Z satisfy the proper conditions. The last of these to come to the writer's attention are two theorems by Vietoris and one by the author of this note.† Such theorems are of course generalizations of Tietze's separation axioms‡ and it might prove profitable to work out systematically the possibilities along this line.

In some of the generalizations mentioned it is shown that, if Z is locally connected, a decomposition $Z = M + N$, where M and N are also locally connected, is possible, but, as far as the writer knows, the following theorem, which shows a certain kind of local connectivity for $M \cdot N$ as well as for M and N, is new.

THEOREM. *Let A and B be sub-continua of the locally connected compact metric space Z and let $A \cdot B$ be totally disconnected or void.*

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† L. Vietoris, *Über den höheren Zusammenhang*, Fundamenta Mathematicae, vol. 19, pp. 271–272; W. A. Wilson, *On unicoherency about a simple closed curve*, American Journal of Mathematics, vol. 55, p. 141.

‡ See F. Hausdorff, *Mengenlehre*, p. 229.

Then there are two locally connected continua M and N such that $Z=M+N$, $M\cdot(B-A\cdot B)=0=N\cdot(A-A\cdot B)$, $A\cdot B\subset M\cdot N$, and each component of $M\cdot N$ is locally connected.

PROOF. Set $\alpha=A-A\cdot B$ and $\beta=B-A\cdot B$. Since A and B are continua and $A\cdot B$ is totally disconnected, $A=\bar{\alpha}$ and $B=\bar{\beta}$. As α and β are mutually separated sets, there are closed sets F and G such that $Z=F+G$, $\alpha\cdot G=0$, and $\beta\cdot F=0$. Let R be the union of the components of $Z-G$ which contain points of α. Then $A\subset\overline{R}$ and so $\overline{R}$ is a continuum. Let S be the union of the components of $Z-\overline{R}$ which contain points of β, and let T be the union of the other components. As in the case of $\overline{R}$, we see that $B\subset\overline{S}$ and $\overline{S}$ is a continuum. Set $H=\overline{R}+T$ and $K=\overline{S}$. Clearly H is a continuum, $Z=H+K$, $\beta\cdot H=0$, $\alpha\cdot K=0$, and $A\cdot B\subset H\cdot K$.

The continua H and K are not necessarily locally connected at points of $H\cdot K$, although they are locally connected at all other points. Set $J=H\cdot K-A\cdot B$. The set J contains no point of $A+B$ and it is the union of a sequence of closed sets $\{J_i\}$ defined as follows. Take a sequence of positive numbers $\{\eta_i\}$ such that $2\eta_{i+1}<\eta_i$, let J_1 be the set of points of J whose respective distances from $A\cdot B$ are not less than η_1, and in general let J_i be the set of points of J whose respective distances from $A\cdot B$ are not more than η_{i-1} and not less than η_i. (Some of these may be void.) We then choose a descending sequence of positive numbers $\{\sigma_i\}$ converging to zero such that each σ_i is less than the distance between J_i and $A+B$. If $\epsilon_i<\sigma_i/3$, there is a finite set of locally connected continua whose union we call L_i such that the points of J_i are inner points of L_i (relative to Z) and $L_i\subset V\epsilon_i(J_i)$.* If also ϵ_i is small enough, no two sets L_i and L_j will have common points unless $|i-j|=1$.

Let L be the union of the sets $\{L_i\}$. If a is a limiting point of L not contained in L, it is in no L_i and is the limit of a sequence of points $\{a_j\}$, where each a_j lies in L_j and j runs over a partial sequence of the numbers $i=1, 2, 3, \cdots$. Since $\epsilon_i\to 0$, $\eta_i\to 0$, and $L_i\subset V\epsilon_i(J_i)$, it follows that a lies in $A\cdot B$. Thus $\overline{L}\subset L+A\cdot B$. Note also that $L\cdot(A+B)=0$.

Let $P=L+A\cdot B$ and C be a component of P. If C has but one point, it is locally connected in a trivial sense. In the other

* By this notation is meant the set of points (of Z) whose respective distances from J_i are less than ϵ_i.

event C is the sum of a sub-set of L and a sub-set of $A \cdot B$. If a is a point of C not in $A \cdot B$, a lies in one L_i only or perhaps in $L_i + L_{i+1}$. In either event there is a $\delta > 0$ for which $P \cdot V_\delta(a)$ lies in a locally connected sub-continuum of $L_i + L_{i+1}$, which in turn is contained in C. But then C is locally connected at *every* point, since $C \cdot A \cdot B$ is totally disconnected and no continuum is locally connected at all points except those of a totally disconnected set. Thus each component of P is locally connected.

Now set $M = H + P$ and $N = K + P$. Clearly $M \cdot N = P$, $M \cdot (B - A \cdot B) = N \cdot (A - A \cdot B) = 0$, and $A \cdot B \subset M \cdot N$. It is obvious that M is locally connected at the points of $M - P$. For any other point x we have $M \cdot V_\epsilon(x) = H \cdot V_\epsilon(x) + P \cdot V_\epsilon(x)$. If x lies in $H - H \cdot K$ and ϵ is small enough, $V_\epsilon(x) \subset H - H \cdot K$, and therefore M is locally connected at x. If x lies in $K - H \cdot K$, then $M \cdot V_\epsilon(x) = P \cdot V_\epsilon(x)$ for ϵ small enough. In consequence of the discussion in the previous paragraph we can also take ϵ so small that $P \cdot V_\epsilon(x)$ lies in some one component C of P and therefore M is locally connected at x on account of the local connectivity of C at x. If x lies in $J = H \cdot K - A \cdot B$, then x is an inner point of some one of the locally connected continua whose union is L, whence M is locally connected at x. Thus M is locally connected at all points except possibly those of the totally disconnected set $A \cdot B$; it is therefore everywhere locally connected. Likewise N is everywhere locally connected and the theorem is proved.

In conclusion it should perhaps be noted that the number of components of $M - N$ depends upon the nature of Z; for example, if Z is further restricted to being unicoherent, then $M \cdot N$ is a locally connected continuum.

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