quickly succeeded by the American Mathematical Society, whose publications made a privately supported journal less necessary. This consoled Professor Stone in giving up the publication of the Annals.

American students in mathematics owe Professor Stone a real debt of gratitude for his earnest effort in helping to create an interest in their subject.

J. J. LUCK

HILBERT'S INTUITIONAL GEOMETRY

Anschauliche Geometrie. By D. Hilbert and S. Cohn-Vossen. Berlin, Julius Springer, 1932. vii+310 pp.+330 figures. R.M. 25.80.

The foundation of this extremely interesting book is a course of lectures delivered by Hilbert at the University of Göttingen in the winter of 1920–21. The notes were taken by W. Rosemann and supplemented and edited by S. Cohn-Vossen.

In the preface Hilbert points out that in mathematics as in all other scientific research we meet two sorts of tendencies: one towards abstraction, the other towards intuition. The first seeks to work out the logical points of view from the extensive material and to connect it systematically; the second strives for an intuitional conception and an understanding of relations of content.

The first has led to the magnificent systematic theories embodied in algebraic geometry, Riemannian geometry, and topology. Nevertheless, Hilbert maintains the position that intuitional geometry is still of great importance as a superior force of research and for the appreciation of the results of research.

The reviewer may be permitted to illuminate this by a personal experience. It is well known that the sextic of genus four, as the intersection of two generically located cubic and quadric surfaces, admits of 120 tritangent-planes. Just as in the case of the 28 double tangents of a general plane quartic, which may all be real, the question arises whether all 120 tritangent-planes may be real. I know of no place where this question has been answered nor of any method by which this problem could be solved. But I constructed a model of this sextic which shows the possibility of 120 real tritangent-planes. This, of course, is no mathematical proof.

Thus, in this, as in many other instances of mathematical research, it is obvious that intuitional aid by the way of construction of graphs and models and intuitional interpretation is in many cases very desirable and helpful.

Throughout the book the reader is struck by the loftiness of the standpoint from which the problems are viewed. One would of course expect this from a superior mathematical mind like Hilbert's.

The first chapter deals with the simplest curves and surfaces, conics and quadrics with their most striking characteristic properties.

In the second chapter are considered regular point systems in the plane and in space. The study of lattice-works has in recent years become of great importance not only for the proper comprehension of crystallographic systems but also for certain branches of number theory as appears from the memorable investigations of Minkowski. This is shown by such examples as the Leibniz

series $\pi/4 = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$, the groups of plane and spatial movements and their application to the classification of crystals, regular polyhedrons, etc.

One of the most fascinating chapters of mathematics, in particular of geometry, is that of configurations and their connection with various geometries and finite group theory. Here, in our book, we find a discussion of the (7,3), (8,3), (9,3)-configurations, the theorems of Desargues and Pascal and their position in projective geometry, the configuration of the centers of four spheres and their centers of similitude, regular solids and cells in four-dimensional space and their projections, an account of enumerative methods in geometry, and Schläfli's double-sixes on the cubic surface. One may expect that differential geometry will play an important part in a book on intuitional geometry. This we find in Chapter IV, under such topics as differential properties of plane and space curves, curvature, developable and ruled surfaces, curves and surfaces of constant breadth, the principal properties of the sphere ("about which a whole book might be written"), cyclides, non-euclidean geometries and some of their models, circular transformations and stereographic projection, the partition of the complex plane by circular arcs and connected groups.

Next follows a chapter on kinematics, dealing with movements in the plane and in space, with Peaucellier's cell and other linkages, and well known curves generated by moving points subjected to certain mechanical conditions.

In Chapter VI we find a very interesting introduction to topology, a branch of mathematics of relatively recent origin, whose importance in the evolution of modern mathematics has become more and more apparent.

The discussion first centers about Euler's theorem and its generalization and the connectivity of surfaces. Then follows a rather detailed study of unilateral surfaces as exemplified by the projective plane and its various models. As such are presented various models constructed at Göttingen, Steiner's famous quartic surface (Römerfläche) and Boy's surface, for which no algebraic equivalent has been found so far.

Among the particularly fascinating problems of topology are those concerned with neighboring regions, as the four-color problem, and the dualistic "thread-problem" (Fadenproblem). As the latter is not as well known it is perhaps of interest to many readers to have it stated: "Given a surface; required the maximum number of points on it which may all be connected to each other by curves on the surface such that no two curves intersect each other." In the popular sense curve is identified with thread. As is well known, the four-color problem, the possibility of coloring any geographic map with four colors so that no two adjacent regions have the same color, has not yet been shown to have a solution in every possible case. It is one of those famous unsolved problems (like some in number theory) whose simplicity is such that any person with ordinary intelligence can grasp its content.

The book is clearly written and beautifully illustrated, and from a didactic standpoint will do a great deal of good. It will be thoroughly enjoyed by analyst and geometrician alike.

<div align="right">ARNOLD EMCH</div>