ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THIS SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

159. Dr. O. K. Bower: *Applications of an abstract existence theorem to both differential and difference equations.*

A functional equation of the form $f=g+Sf$, in which g is a known function and S a suitable functional operator, has for some time been used to prove existence theorems for integral and differential equations (Max Mason, *Selected topics in the theory of boundary value problems of differential equations*, The New Haven Mathematical Colloquium, 1906, p. 173). This paper considers an equation of this form with modified restrictions on the function g and operator S and proves an abstract existence theorem which can be applied directly to proving the existence of solutions of integral, finite integral, q-integral, differential, difference, and q-difference equations having relatively simple solutions in the neighborhood of infinity. The principal novelty of this treatment lies in making all of the six cases depend on a single abstract theorem, which is possible because of the inherent similarities in the operations of differentiating, differencing, and q-differencing. (Received May 11, 1932.)

160. Dr. O. K. Bower and Mr. J. D. Grant: *A system of simultaneous bilinear functional equations.*

This paper considers the system of simultaneous bilinear functional equations S: $u_j(x+y)=\sum_{i=1}^{n} a_{i,\, n+j+1-i}u_i(x)u_{n+j+1-i}(y)$, $j=1, 2, \cdots, n$; $a_{i,\, n+j+1-i}=a_{i,\, j+1-i}$, $u_{n+j+1-i}=u_{j+1-i}$, $i\leqq j$, in which the constant coefficients $a_{i,\, n+j+1-i}$ are such that there is a solution consisting of n linearly independent and continuous functions. This system of equations is a generalization of the cosine and sine, hyperbolic cosine and hyperbolic sine addition formulas ($n=2$), and has likewise as a special case when $n=3$ the functions u_1, u_2, u_3 discussed by Mr. L. E. Ward (American Mathematical Monthly, vol. 34 (1927)). It is shown that there are as many "normal" forms for S for each value of n as the number of divisors of n, unity and n included, and that the number of entirely arbitrary constants in the general solution of each of these forms is equal to n. (Received May 11, 1932.)

161. Professor Alonzo Church: *A set of postulates for the foundation of logic.* Second Paper.

In a recent paper (Annals of Mathematics, vol. 33 (1932), pp. 346–366) the author proposed a set of postulates for logic, which, it was hoped, would avoid some of the complications of *Principia Mathematica*. In the present paper it is found necessary to add four new postulates to this set, and it is also observed that Postulates 2 and 24 of the original set are not independent and may be omitted. Some of the first consequences of the revised list of postulates are developed. It is proved that, of all propositional functions equivalent to ϕ, there is one, called the *completion* of ϕ, for which the range of the independent variable is greatest. In terms of the notion of the completion of a propositional function, theorems are proved which validate certain forms of the method of reductio ad absurdum. A program is outlined by which it is hoped to obtain a theory of positive integers and a theory of real numbers. The possibility is discussed of making a metamathematical proof, in the sense of Hilbert, that these postulates cannot lead to a contradiction, and it is pointed out that the recent proof by Kurt Gödel (Monatshefte für Mathematik und Physik, vol. 38 (1931), pp. 173–198) of the impossibility of such a proof of freedom from contradiction does not apply to this case. (Received May 16, 1932.)

162. Mr. J. G. Deutsch: *Functions satisfying a generalized Lipschitz condition.*

In this paper functions are defined on a set E which satisfy a Lipschitz condition relative to sets whose measures approximate that of E. Such a function is said to have the L-property relative to E. A necessary and sufficient condition is established that a function have the L-property. For example, functions which possess finite partial derivatives have the L-property. Transformations whose components have the L-property are considered and certain of the classical results on transformations are shown to be valid for these transformations. It is proved that a function of two or more variables which is monotone on a set is totally differentiable relatively to that set almost everywhere in the set. It is a consequence of this result that functions of bounded variation in the sense of Arzelà or in the sense of Hardy and Krause are totally differentiable almost everywhere. (Received May 13, 1932.)

163. Mr. C. A. Lovell: *On certain associated metric spaces.*

For every Einstein space of non-zero scalar curvature there exists an associated space whose fundamental tensor is the Ricci tensor for the Einstein space. This paper considers, among other things, the space defined by the assignment of a value to the curvature of the associated space. The idea of associating surfaces in the manner mentioned above is extended to produce an ordered sequence of surfaces, the fundamental tensor of each member of the sequence being the Ricci tensor of the surface which precedes it in the sequence. The members of the sequence are the original surface, the first associated Ricci surface, the second associated Ricci surface, etc. Among the interesting special cases discussed are those in which the first associated space is flat or has constant scalar curvature. Included in one of these special cases are found to be all surfaces applicable to a minimal surface. The equation of condition for these cases is integrated, giving the most general examples of such surfaces. The case where the mth associated Ricci surface is the original surface is also discussed. (Received May 24, 1932.)

164. Professor A. A. Albert: *Non-cyclic normal division algebras of degree and exponent four.*

The author recently proved the existence of non-cyclic normal division algebras in a paper to be published in the June, 1932, issue of this Bulletin. The algebras constructed were direct products of (cyclic) algebras of order four and hence did not have a structure very different from cyclic algebras. The present paper presents certain normal division algebras of order sixteen not expressible as direct products of algebras of order four and proves them to be non-cyclic. Thus the first example in the literature is given of normal division algebras with a structure proved to be more complicated than that of the simplest type, the cyclic (Dickson) algebras. (Received May 26, 1932.)

165. Miss Esther McCormick: *On reduction of space Cremona involutions.*

Every involutorial transformation of the second degree can be reduced. Let T_m be any one of the general monoidal involutions with stars coincident and corresponding in homology. If an S_n exists such that $S_n^{-1}T_mS_n$ is a collineation, then $n>m$ and $n\geqq am$, where a depends on an F curve of S_n. For every m there are infinite sets of T_m's which can be reduced to collineations by an S_n with $n<m$. (Received June 3, 1932.)

166. Professor H. R. Brahana: *Groups $\{S, T\}$ whose commutator subgroups are abelian.*

This paper gives a complete determination of the groups generated by two operators, T of order 2 and S of prime order p, whose commutator subgroups are abelian. (Received June 14, 1932.)

167. Professor H. R. Brahana: *Prime-power abelian groups generated by a set of conjugates under a special automorphism.*

The abelian groups H which are commutator subgroups of groups $\{S, T\}$ have, among others, the following properties: H admits an isomorphism U of order p, the order of S; H is generated by the conjugates of one of its operators under powers of U, of which no more than $p-1$ are independent; H contains no operator of order different from p invariant under U. If H is of order q^n, where q is a prime distinct from p, then $n=\alpha(k_1m_1+k_2m_2+\cdots+k_jm_j)$ and H has $k_i\alpha$ independent generators of order q^{m_i}, where α is the exponent to which q belongs, mod p, and $k_1+k_2+\cdots+k_j\leqq(p-1)/\alpha$. If H is of order p^n, then $n=k_1m+k_2(m-1)$ and H has k_1 and k_2 independent generators of order p^m and p^{m-1} respectively, where $k_1+k_2=p-1$ and $m\geqq 1$; or $n=k+2$ and H is of type $2, 1, 1, \cdots$, where $k<p-2$. (Received June 14, 1932.)

168. Dr. Leonard Carlitz: *Note on polynomials in a Galois field.*

In this paper are proved a number of miscellaneous results concerning polynomials in a single indeterminate with coefficients lying in a Galois field. The results are of two kinds. The first are formulas involving certain divisor and generalized totient functions. The second consist of expressions for the

L.C.M. of the polynomials of given degree, the product of all such polynomials, and the product of those that are not divisible by any ρth power ($\rho>1$). (Received June 16, 1932.)

169. Dr. Leonard Carlitz: *On the representation of a polynomial in a Galois field as the sum of an even number of squares.*

In this paper simple expressions are obtained for the number of solutions of $\alpha F=\alpha_1 X_1^2+\beta_1 Y_1^2+\cdots+\alpha_s X_s^2+\beta_s Y_s^2$, where(A): $\alpha=\alpha_1+\cdots+\beta_s\neq 0$, $(\alpha_1+\beta_1)\cdots(\alpha_s+\beta_s)\neq 0$; X_i, Y_j of degree k, F of degree $2k$; (B): $\alpha_1+\cdots+\beta_s=0$; X_i, Y_j of degree k, F of degree $f<2k$. The results hold for all positive s, the form depending essentially on whether $(-1)^s\alpha_1\cdots\beta_s$ is or is not a square in the underlying Galois field. (Received June 16, 1932.)

170. Dr. Leonard Carlitz: *On the representation of a polynomial as the sum of an odd number of squares.*

By means of the results in a paper on an even number of squares (this Bulletin, vol. 38 (1932), abstract 169), the corresponding problems are here treated for the case of an odd number of squares. The results are different from those for the former problem; they involve the sums $\sigma_t=\sum(A/F)$, summed over all A of degree t, (A/F) being Dedekind's symbol of quadratic residuacity. For the case F quadratfrei, the results are particularly simple. (Received June 16, 1932.)

171. Professor R. D. Carmichael: *Systems of linear difference equations and expansions in series of exponential functions.*

The principal purpose of the first part of this paper is to establish existence theorems for the solution of the system $\sum_{j=1}^{n} e_{\nu j} g_j(x+a_{\nu j})=\phi_\nu(x)$, $(\nu=1, 2, \cdots, n)$, of generalized non-homogeneous linear difference equations with constant coefficients, the given functions $\phi_\nu(x)$ being integral functions and the dependent functions $g_j(x)$ being required to be integral functions. The results obtained are applied in the second part to the rather remarkable problem of the simultaneous expansion of n given integral functions in composite power series. Finally the third part of the paper is devoted to the theory of a class of remarkable expansions in series of exponential functions, generalizing the theory of Fourier series in an interesting direction. Whereas the basic region of convergence of Fourier series is a segment of a straight line, these new series, apart from certain particular cases, have certain polygons in the complex plane as their basic regions of convergence. (Received June 18, 1932.)

172. Dr. A. E. Currier: *The fundamental theorems on second-order cross partial derivatives.*

In a paper submitted to the Transactions of this Society we have proved the following theorems. Theorem 1. *Let $f(x, y)$ be a function of Baire defined on an open region R. Let the first partial derivative f_x exist on R. Let A be a point set on which the partial derivatives f_{xx} and f_{xy} exist almost everywhere. Then the following* *approximate limit*, approx-$\lim_{\lambda\to 0}(1/\lambda^2)[(x+\lambda, y+\lambda)-f(x, y+\lambda)-f(x+\lambda, y)+f(x, y)]$, *exists and equals* f_{xy} *almost everywhere on* A. Theorem 2. *Let* $f(x, y)$ *be defined on an open region* R. *Let* f_x *and* f_y *exist on* R. *Let* A *be a set on which* f_{xx}, f_{xy}, f_{yx}, f_{yy} *exist almost everywhere. Then* $f_{xy}=f_{yx}$ *almost everywhere on* A. We can replace f_x by D_xf in the above theorems, where D_xf is the upper or lower right or left hand first partial derivative of f with respect to x. Theorems 1 and 2 can now be generalized so as to avoid all reference to the existence of first partial derivatives. (Received June 9, 1932.)

173. Mr. H. G. Russell and Professor J. L. Walsh: *On the convergence and overconvergence of sequences of polynomials of best simultaneous approximation to several functions analytic in distinct regions.*

Let M be an arbitrary closed limited point set (not a single point) of the z-plane whose complement K (with respect to the entire plane) is connected and regular in the sense that the Dirichlet problem can be solved for K. Let $w=\omega(z)$ be a function which maps K onto the exterior of the unit circle in the w-plane so that $z=\infty$ corresponds to $w=\infty$. Let C_R denote the transform in the z-plane of $|w|=R$, $R>1$, under the mapping function $\omega(z)$. Then it is proved that (1) if the function $f(z)$ is analytic on and within C_R, there exist polynomials $P_n(z)$ of respective degrees $n=1, 2, \cdots$, such that the inequalities (a) $|f(z)-P_n(z)|\leqq N/R^n$, z on M, where N is independent of n and z, are valid for z on M; (2) if there exist polynomials $P_n(z)$ of degrees $n=1, 2, \cdots$, such that (a) holds for z on M, the sequence $\{P_n(z)\}$ converges uniformly on any closed point set interior to C_R. Applications are made to sequences of polynomials of *best approximation.* (Received May 25, 1932.)

174. Professor J. P. Ballantine: *A proof of E. H. Moore's equivalence.*

Moore's equivalence states that the existence of a solution of a system of linear equations is equivalent to a certain implicational proposition. His proof holds in any number system where division except by zero is always uniquely possible. In order to orthogonalize the columns of the matrix of the coefficients, he has to assume that if any column, considered as a vector, has a norm zero then every coefficient in that column vanishes. In a system of residues modulo p, important cases arise when this hypothesis is not satisfied, and yet the equivalence holds. A proof is given which does not require the hypothesis. It is pointed out that the fundamental lemma of the calculus of variations is a special case of Moore's equivalence. (Received June 20, 1932.)

175. Mr. Amos Black: *Types of involutorial space transformations associated with certain rational curves.*

Let there be given a pencil of surfaces $|F_n|$ which contains a rational curve r_m as an $(n-2)$-fold basis curve. If the surfaces of the pencil are projective with the points of the curve, a general line of the complex of secants of r_m will intersect the associated surface in two points P, P' not on r_m. These two points are a pair in involution. The admissible cases other than the straight line, n general, m general, and the pencil of quadrics are: $n=3$, $m=2, 3, 4, 5$; $n=4$, $m=2, 3$. When $n=4$, $m=3$ the involution has a new kind of singularity, an infinity of parasitic lines which are the generators of a ruled surface. (Received June 22, 1932.)

176. Dr. A. L. Foster: *On general Kronecker-(integer)-synthesis of disciplines.*

The present communication establishes certain foundations basic in any integer-synthesis, or analysis, in the Kronecker sense; i.e., in the construction (or reduction) of mathematical disciplines as integer-disciplines. (Cf. J. Stein, Mitteilen der Mathematischen Gesellschaft in Hamburg, vol. 7, No. 2, in which the disciplines (1) rational arithmetic, and (2) (Gauss) complex number arithmetic are Kronecker-reduced in a satisfactory way.) The construction and study of (1) equivalence-functions (e.g., the "identical," $n=n$ of arithmetic; or again, for example, the "derived" equivalence, $\asymp$, which says $m/n \asymp m/n$ if $mn=mn$), and (2) equivalence-conserving functions (essentially functions which, for equivalent arguments, take on equivalent values) are considered. In (1) a general and powerful method of mathematical construction of equivalence functions is discovered, in a covering process, associated with Abelian systems (not necessarily groups) of transformations. The development proceeds from an extremely primitive axiomatic. ("Objects," "characteristics" (or "Merkmale"), "basic equivalence function.") As yet merely certain simple special "denumerable" cases opened up by this theory have been considered. Besides familiar disciplines, other cases present themselves. To what extent this type of investigation will also prove fruitful in the "non-denumerable" case cannot as yet be said. (Received June 23, 1932.)

177. Mr. Ralph Hull: *The numbers of solutions of congruences involving only k-th powers.*

Following preliminary theorems concerning the numbers of solutions of general congruences of the type $\sum_\nu a_\nu x_\nu{}^k \equiv a \pmod{n}$, ($\nu=1, \cdots, s$; $k \geqq 1$; $s \geqq 1$; $a_1, \cdots, a_s$, a and n any integers), the congruence $\sum_\nu x_\nu{}^k \equiv a \pmod{p}$, p a prime, is considered. This congruence has the same number of solutions as the congruence $\sum_\nu x_\nu^m \equiv a \pmod{p}$, where m is the greatest common divisor of k and $p-1$, and formulas are found for the numbers of solutions for $m \geqq 2$. These are of the nature of recursion formulas and reduce the problem for given m and p to the determination of m^2 integers for obtaining which a method is indicated. For $m=5$, the general formulas, together with a special discussion, yield expressions for the coefficients of the reduced form of the quintic resolvent of the cyclotomic equation $x^{p-1}+\cdots+1=0$, $p=5h+1$, in terms of an integral solution of the simultaneous equations $x^2+25y^2+25z^2+125w^2=16p$, $y^2+yz-z^2=xw$. These equations have exactly eight distinct solutions in integers for each prime $p=5h+1$. Sufficient conditions on s are indicated in order that the congruence $\sum_\nu x_\nu^k \equiv a \pmod{n}$ may have a solution for every a and n. (Received June 24, 1932.)

178. Professors W. D. MacMillan and Walter Bartky: *Permanent configurations in the problem of four bodies.*

If there exists a configuration in the problem of n bodies in which the attraction on each one of the bodies due to all of the others is directed toward the center of gravity and is proportional to the distance of the body from the center of gravity, then the equations of motion of the n bodies reduce to the equations of motion of the two-body problem. Therefore Keplerian motion of the n bodies, in which the configuration is preserved, is possible. The Lagrangian solutions for three bodies, Moulton's straight line solution for n bodies, and other particular configurations come under this general theorem. Hence the problem reduces to finding such configurations. An analysis of the configurations of the four-body problem is given. Quadrilaterals are classified: convex, concave, and straight lines. By Moulton's theorem, any four masses in any assigned order can be arranged uniquely on a straight line so as to satisfy the necessary conditions. For any four masses and assigned order there exists at least one convex quadrilateral (uniqueness not proved) that satisfies the conditions. For concave quadrilaterals the same theorem holds provided the mass ratios lie within certain bounds. Within certain sharp restrictions the quadrilateral can be taken at random, and the masses are, with a single exception, uniquely determined. These methods extend to the case of more than four bodies. (Received June 23, 1932.)

179. Professor R. D. Carmichael: *Note on triple systems.*

The theorem of Skolem (Norsk Matematisk Tiddsskrift, vol. 13 (1931): pp. 41–51), proved also by Hasse (ibid., pp. 105–107), concerning the existence of triple systems having the property that the presence of the triples (abu), (bcv), (ucx) in such a system implies the presence of the triple (avx) in the system is easily shown to be essentially a special case of the existence theorem for the finite geometries of Veblen and Bussey: the triples in such a system are the sets of collinear points in the geometry $PG(k, 2)$ where k is any positive integer. (Received June 25, 1932.)

180. Professor W. H. McEwen: *On the approximate solution of linear differential equations with boundary conditions.*

This paper is concerned with the approximate representation of the solution of a linear differential equation of the mth order $L(y)=R(x)$ with m linearly independent two-point boundary conditions $U_i(y)=h_i$ $(i=1, 2, \cdots, m)$, by means of polynomials $P_n(x)$ defined so as to minimize an expression involving the boundary conditions as well as the integral of the rth power of $|L(P_n)-R|$. (Received June 27, 1932.)

181. Professors E. R. Hedrick and W. M. Whyburn: *An application of the Dedekind cut notion to integration.*

The authors introduce a notion of *cut,* analogous to the Dedekind number cut, in the class of all simple functions on the real interval $a\leqq x\leqq b$ (see this Bulletin, vol. 38, page 123, for definition of *simple function*). Two classes of functions are defined on a basis of this cut. One of these classes contains all bounded functions that are Riemann integrable and is contained in the class of all Lebesgue integrable functions. It is shown to be identical with the class of all functions that are continuous on the given interval minus a set of points of measure zero. The second class contains all functions (bounded or unbounded) that are integrable in the Lebesgue sense. Other developments of the notion of *cut* are given. (Received June 27, 1932.)

182. Professor E. V. Huntington: *New sets of independent postulates for the algebra of logic, with special reference to Whitehead and Russell's Principia Mathematica.*

This paper contains two sets of postulates for Boolean algebra in terms of $(K, +, ', =)$, and the following set of nine postulates for Section A of the *Principia* in terms of $(K, +, ')$ alone. *Group A*. 1.71. If a and b are in K, then $a+b$ is in K. 1.7. If a is in K, then a' is in K. *Group B*. There exists in K a subclass T having the following five properties: 1.1. If a is in T and $a'+b$ is in T, then b is in T. 1.2. $(a+a)'+a$ is always in T. 1.3. $b'+(a+b)$ is always in T. 1.4. $(a+b)'+(b+a)$ is always in T. 1.6. $(b'+c)'+[(a+b)'+(a+c)]$ is always in T. *Group C*. If T is a subclass having these five properties, then T has the following further properties: 1.8. If $a+b$ is in T, then a is in T or b is in T. 1.9. If a' is in T, then a is not in T. The first seven postulates correspond to the "formal" primitive propositions in the *Principia*; the last two correspond to two "informal" statements in the *Principia*. Through postulates 1.8 and 1.9 the "formal" and "informal" definitions of $a \equiv b$ (as an "element" and as a "relation") become equivalent. The consistency and independence of the nine postulates are established by usual methods. (Received June 25, 1932.)

183. Professor Nathan Altshiller-Court: *The Apollonian spheres of a tetrahedron.*

Consider the four spheres having for centers the vertices of a tetrahedron (T) and having their radii proportional to the respective altitudes of (T). The six spheres of similitude of these four spheres, taken in pairs, are, by definition, the *spheres of Apollonius* of the tetrahedron. Apollonian spheres exhibit many analogies to similarly defined Apollonian circles of a triangle. The two bisecting planes of a dihedral angle of (T) meet the opposite edge in two diametrically opposite points of the Apollonian sphere having its center on the latter edge. The centers of the six Apollonian spheres lie in the same plane, the *Apollonian plane* of the tetrahedron. The six Apollonian spheres have two points in common, but, contrary to the case of the Apollonian circles, these two points are not necessarily real. The six radical planes of the Apollonian spheres with the circumsphere of (T) have a point in common. The harmonic pole of the Apollonian plane with respect to (T) and the centroid of (T) are two isogonal conjugate points of (T). The twelve points determined by the pairs of bisecting planes of the dihedral angles of a tetrahedron upon the respectively opposite edges form a desmic system of tetrahedrons. (Received July 5, 1932.)

184. Mr. G. P. Wadsworth: *Systems of three linear partial differential equations of the third order.*

This paper considers the solution of a system of three linear homogeneous partial differential equations of the third order, using an extension of the method of P. Franklin and C. L. E. Moore (Journal of Mathematics and Physics, vol. 9, No. 1, 1930, p. 22) for two partial differential equations of the second order. Two canonical forms are found to which the original equations can be reduced, and it is shown how the three canonical forms which E. P. Lane (The Tôhoku Mathematical Journal, vol. 33, Nos. 1, 2, p. 12) found from geometrical considerations may be reduced to these two forms. Thus the solution of a system of third order linear partial differential equations reduces to the solution of one of the two canonical forms of which it is a consequence. In the case of one of the canonical forms, the solutions are linear combinations with arbitrary constant coefficients of eight or fewer definite functions. In the case of the other canonical form, the solutions are certain linear combinations of arbitrary constants, an arbitrary function, and its derivatives, where the coefficients of these functions are definite functions which depend upon the original system of equations. (Received July 1, 1932.)

185. Professor W. M. Whyburn: *A classification of the critical sets for functions.*

In a former paper (this Bulletin, vol. 35, pp. 701–708) the author studied the *critical sets* for continuous real functions of n real variables. The functions were assumed to have continuous first partial derivatives. The present paper separates the critical sets of functions into five classes on the basis of the complimentary domains of certain closed point sets. A notion of *order* of a critical set is introduced and relations are established between the five classes. Four of the classes correspond, in a general way, to the ordinary minimal, maximal, minimax, and flex points and degenerate into these under more restrictive hypotheses. The notions of *critical point* and *critical set* are extended to continuous functions which do not have first partial derivatives, and many of the former results are shown to hold for this less restricted class of functions. (Received July 6, 1932.)

186. Professor E. T. Bell: *Diophantine equations from algebraic invariants and covariants.*

Few Diophantine equations of degree higher than the second in more than two variables have been *completely* solved in integers; see Dickson, this Bulletin, vol. 27, 1921, pp. 312–319, also *History of the Theory of Numbers*, vol. 2. In the present paper the following method is applied to obtain *complete* solutions of some such equations. Necessary and sufficient conditions that a given general quantic have a repeated factor, or that it be totally reducible, or that it have a factor of given degree, etc., are expressed by the *identical* vanishing of certain covariants. Equivalent conditions can be obtained by the method of undetermined coefficients. Comparing the two sets, we get a *complete parametric solution* of a Diophantine system. From this solution, *all integer solutions* are obtained by selecting from the sets of values, rational and irrational, of the parameters *all* those which give integer points on the locus represented by the system. The method is applied in detail to the general quartic developable in homogeneous coordinates in 3-space, and explicit formulas in terms of integer parameters are given for all integer points on the surface. (Received July 7, 1932.)

187. Mr. H. W. Raudenbush, Jr.: *Differential fields and ideals of differential forms.*

Fields in which each element has a unique derivative which is also an element have been considered by R. Baer and used by O. Ore. In this paper, the author shows that extensions of such differential fields to differential fields satisfy theorems analogous to Steinitz' theorems (Journal für Mathematik, vol. 137 (1910), p. 167; theorem 2, p. 293; 3, p. 299; 4, p. 300) on the degree of transcendency of extensions of algebraic fields. A differential form is defined as a polynomial in indeterminates and their derivatives, with coefficients elements of a given differential field. For such differential forms, it is shown that there exists a theory analogous to a part of van der Waerden's theory (Mathematische Annalen, vol. 96 (1926), p. 183; in particular, pp. 191–192) of ideals of polynomials. (Received July 7, 1932.)

188. Dr. James Singer: *Three-dimensional manifolds and their Heegaard diagrams.*

In this paper the problem of the classification of 3-dimensional manifolds is reduced to the problem of the classification of Heegaard diagrams. Heegaard, in a paper republished in the Bulletin de la Société Mathématique de France, vol. 44 (1916) (see also Veblen, *Analysis Situs*) showed that a 3-dimensional manifold may be represented by a diagram consisting of a closed and connected 2-dimensional manifold upon which are drawn two sets of "canonical" curves. Unfortunately, a manifold may give rise to an infinity of distinct diagrams. We define a set of "moves" which transform a diagram into another, and then say that two diagrams are equivalent if one can be transformed into the other by a finite number of these moves. It is then proved that any two diagrams arising from manifolds equivalent in the sense of semi-linear analysis situs are equivalent, and conversely, equivalent diagrams give rise to manifolds equivalent in the sense of semi-linear analysis situs. (Received June 17, 1932.)

189. Professor Solomon Lefschetz and Dr. J. H. C. Whitehead: *On analytic complexes.*

In his Colloquium Lectures (p. 364) Lefschetz outlined a proof of the theorem according to which an analytic locus can be covered with a simplicial complex. The object of the present paper is to give a detailed proof along the lines there indicated. See also van der Waerden (Mathematische Annalen, vol. 103, pp. 337–362), Koopman-Brown (Transactions of this Society, vol. 34, pp. 231–252). (Received July 8, 1932.)

190. Professor A. A. Albert: *A note on normal division algebras of order sixteen.*

The author recently proved the existence of non-cyclic normal division algebras of order sixteen and showed that his determination of all normal division algebras (Transactions of this Society, vol. 31 (1929), pp. 253–260) gave the best possible result. The determination was long and complicated and a better proof is desirable. The purpose of this short note is to present such a proof. (Received July 14, 1932.)

191. Professor Dunham Jackson: *Orthogonal trigonometric sums.*

This paper extends to the case of trigonometric sums certain properties of systems of polynomials orthogonal over an interval with respect to a given weight function. (Received July 16, 1932.)