FISHER ON INTEREST

The Theory of Interest as Determined by Impatience to Spend Income and Opportunity to Invest it. By Irving Fisher. New York, The Macmillan Company, 1930. xxvii+566 pp.

This book was begun as a revision of the author's *The Rate of Interest* (1907) which has long been out of print. In the *Preface*, Professor Fisher remarks that his theory of interest has scarcely been altered, but its exposition has been so amplified and recast that it may seem more changed to those who misunderstood his first book than it does to him. The result of this thorough revision has been a new book, a complete rewriting of the former book with additions of new material.

Although scarcely what a mathematician would call a book on applied mathematics, this work contains numerous applications of algebraical and graphical methods and has a distinct mathematical tone throughout. A mathematician reading the book will soon become aware of the fact that the author is devoting the first ten chapters to a description of a mathematical model. This model he reproduces in the next four chapters. Such a reader may think that Professor Fisher has chosen a long way around, but he is writing to a larger audience. On p. 312 he remarks "if I were writing primarily for mathematically trained readers, I would have reversed the order, giving the first place to the formulas, following these with the charts for visualization purposes and ending with verbal discussion."

Professor Fisher's theory of interest is, as he states, p. ix, an enlargement and synthesis of already existing partial theories, and is based on *investment opportunity, human impatience* (preference for comparatively early income over comparatively remote or deferred income) and *market exchange* (buying and selling). The assumptions underlying his general theory are as follows.

I. *Investment Opportunity*, (a) each individual has a choice within limits of different income curves $y(t)$ where t is time, and to each curve is associated a risk; (b) the individual selects the income curve which produces a maximum present value, where the present value takes into account the risk element.

II. *Human Impatience*, (a) the degree of impatience of any individual depends upon his income stream, $y(t)$, as chosen by him and modified by exchange; (b) each person, after or while first choosing the option of greatest present value, will then modify it by exchange so as to convert it into that form most wanted by him.

III. *Market Exchange*, (a) the rate of interest must equalize supply and demand, and (b) the expected present value of all loans equals the present value of the borrowings, but due to risk there may be a wide discrepancy between the actual realization and the original expectation.

Hypotheses Ib and IIb are best handled by means of the calculus, but because of the unprepared state of most expected readers, the author relegates the derivation of the necessary conditions to an appendix. Incidentally it may be remarked here that in all previous somewhat similar books the author has consigned all mathematics to appendices.

In the first eight chapters and in the mathematical treatment (Chapters XI–XIV) Professor Fisher assumes that the probability of risk is zero. In Chapter XIV he remarks: "To attempt to formulate mathematically in any useful, complete manner the laws determining the rate of interest under the sway of chance would be like attempting to express completely the laws which determine the path of a projectile when affected by random gusts of wind." He does, however, give a qualitative discussion of what would happen if risk were taken into account. It would seem that by assuming a law of risk obtained from empirical data in a way similar to the way in which a mortality table is drawn up, and by writing the demand as a functional defined on a range extending from some time in the not too remote past to a time in the future as I have indicated in my paper in the Journal of Political Economy, Oct. 1927, one might formulate a mathematical theory of interest which would take risk into account. The question of whether or not such an extended theory would be worthwhile from a practical standpoint would of course remain open.

In the mathematical treatment the author assumes that all of the income for each year is concentrated at the middle of the year (p. 288). If he had not made this restrictive assumption, the problem would have become one in the maxima of functionals instead of one in the maxima of functions. One wishing to develop further the theory of interest from this point of view may find meager hints in a related theory in my papers in this Bulletin, March–April, 1928, and the American Journal of Mathematics, January, 1928.

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TIMOSHENKO ON STRENGTH OF MATERIALS

Strength of Materials. By S. Timoshenko. Two volumes. New York, Van Nostrand Company, 1930. xii+368 pp; ˙ .+401–735 pp.

The recent appearance of these texts on elasticity will be welcomed by engineering colleges and technical schools for many reasons. Perhaps foremost is the fact that it is written in English and fills a large gap between the two extremes of required courses of strength of materials in our engineering colleges and the comprehensive treatise by A. E. H. Love, *The Mathematical Theory of Elasticity*, now in its fourth edition.

Today it is becoming increasingly more difficult for the designer to meet the needs of greater working stresses, as required by more rapidly moving machine elements, unless analytical methods replace "rules of thumb." To further this end the author has provided a mine of interesting methods both for the student in the classroom and the research man in the laboratory. To quote the author's preface: . . . . "At the present time a decided change is taking place in the attitude of designers towards the application of analytical methods in the solution of engineering problems . . . . . The importance of analytical methods combined with laboratory experiments in the solution of technical problems is becoming generally accepted . . . . . It is the aim of this book (vol. I) to present problems such that the student's attention will be