Vorlesungen über Natürliche Geometrie. By Ernesto Cesàro. German translation by Gerhard Kowalewski. Berlin, B. G. Teubner, 1926. Second edition. vi+352 pp.

This is a reprint of the first edition of the German translation published in 1901. The new feature is a valuable appendix by Kowalewski on *Generalized Natural Geometry*. Although extremely short, occupying only pages 332 to 342, this gives a clear and interesting account of the important theories founded by Georg Pick in 1906 and later developed by some of his students and also by Kowalewski.

Natural or intrinsic geometry, in its original form, as studied by Cesàro, deals with the differential invariants of the group of euclidean motions. The generalization made by Pick is to a very large category of continuous groups, including for example the projective and inversion groups. For each r-parameter group G_r in the plane (assuming a certain transitivity condition) a differential invariant of lowest order J is found (the analog of curvature), and also an integral invariant $s=\int \omega dx$ (the analog of arc length). The natural equation of a curve, relative to the given group, is then of the form $\phi(J, s)=0$. The curves defined by $J=$constant are the analogs of the W or anharmonic curves of projective geometry. Kowalewski has discussed the more general category $J=as+b$ in the Leipziger Berichte, 1924.

Pick's fundamental discovery of covariant coordinates is the final topic. The extension to space of three dimensions, which he has treated in his lectures, is still unpublished.

The terms *intrinsic* geometry and intrinsic equation, used by the Italian and English writers, seem more suggestive than *natural* geometry and natural equation, as used by the German school. The present reviewer introduced the term *natural* family of curves (Transactions, 1909, Princeton Colloquium Lectures, 1913) in another connection, namely in the theory of dynamical trajectories and optical rays. The simplest definition is the extremal families defined by $\int Fds=$ mimimum, where F is any point function and ds is the element of length. (Recently Schouten has called such families conform-geodesic.) It would be interesting to generalize this theory of natural families to other groups, ds being used then in the sense of Pick, and the reviewer expects to study this generalization.

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Premières Leçons de Géométrie Analytique et de Géométrie Vectorielle à l'Usage des Élèves de la Classe de Mathématiques et des Candidats aux Grandes Écoles. By E. Lainé. Paris, Vuibert, 1929. 46 pp.

The students who begin their study of the subject with this little book will hardly find it too easy. The exposition is rather condensed and the number of exercises (23) is not sufficient to illustrate various topics treated in the text, of which some, as for instance, the theory of the central axis, perhaps should not belong to the "first lessons" on the subject. The book contains four chapters: Chapter 1. Scalars, vectors, coordinates. Chapter 2. Elements of vector calculus. Chapter 3. Analytic geometry, surfaces and lines, plane and straight line. Chapter 4. Systems of sliding vectors ("vecteurs glissants"). On the whole, it produces an impression of an outline rather than of a textbook.

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