The Foundations of Euclidean Geometry. By Henry George Forder. xii+349 pp. Cambridge University Press, 1927.

Rarely is the object of a book so tersely expressed on the wrapper as in the case of the foundations of euclidean geometry. The publishers quote these words from the preface: "Although the euclidean geometry is the oldest of the sciences and has been studied critically for over two thousand years, it seems there is no text-book which gives a connected and rigorous account of that doctrine in the light of modern investigations. It is hoped that this book will fill the gap."

There may be a difference of opinion on the question of the completeness of the gap filling, but Mr. Forder has written an interesting book and one which should be welcomed by those interested in seeing the rigor of modern analysis achieved in geometrical studies.

Being anxious to achieve exactness, the author intersperses his definitions and axioms with copious notes explaining the advantages of particular statements. Thus, for example, he uses forty pages to establish his concepts of classes, relations, linear order, non archimedean systems, etc.

With this background he proceeds to set up axioms of order, which he points out were "almost completely ignored by Euclid, but which are fundamental for this work."

Starting with point and order as undefined entities definitions are made of line, plane, and space followed by development of fundamental properties of these.

"We have now laid the foundations of geometry. Our Axioms OI–VIII (order axioms) together with a continuity axiom and a Euclidean parallel axiom enable us to erect a complete Euclidean Geometry. Also—they enable us to introduce algebraic methods."

Following the introduction of angles and their properties, the question of congruence is considered. Here are first introduced the congruence axioms. With the introduction of circle axioms it is shown that the congruence axioms may be greatly weakened.

The introduction of the parallel axiom is deferred to the sixth chapter and is then stated in not fewer than eight different forms. In this chapter is also considered the relations between projective and euclidean spaces, ideal elements are defined and the addition of these to euclidean elements is discussed.

The Desargues and Pappus theorems come in for lengthy discussion from various points of view, while constructions are given a chapter to themselves with special attention to the Poncelet-Steiner and Mascheroni theorems and to the Gaussian theory of the regular polygon.

After chapters on analysis situs of polygons, areas of polygons and volumes of tetrahedra the author says: "We now reach the climax of our investigation. So far we have not assumed that our lines are continua. We shall now secure this property by assuming a continuity axiom." He completes his argument by showing that all euclidean geometry may be logically built upon order axioms, a very restricted parallel axiom and a weakened continuity axiom.

The last two chapters are quite distinct, almost in the nature of an appendix "designed to illustrate one or two interesting points" such as congruences as the sole undefined relation between points, non-euclidean areas, etc.

Mr. Forder has undertaken a difficult task and, while his work may be open to criticisms in some respects, he has made a readable book, the study of which by our teachers of euclidean geometry might go far toward eliminating the inaccuracy of concepts and illogical deductions from the minds of those supposedly engaged in teaching accurate, precise thinking. It is a worth while book which may well serve as a stimulus to others to write into our literature more exact presentations of our basic geometric concepts.

F. W. OWENS

Operational Methods in Mathematical Physics. By Harold Jeffreys. London, Cambridge University Press, 1927. vii+101 pp. Price 6 s. 6 d.

This valuable monograph is number 23 of the important series entitled Cambridge Tracts in Mathematics and Mathematical Physics. The author says in the preface: "My own reason for writing the present work is mainly that I have found Heaviside's methods useful in papers already published, and shall probably do so again soon, and think that an accessible account of them may be equally useful to others." As a matter of fact he has filled admirably a gap of some thirty years standing in the literature pertaining to the solution of the differential equations of physics, since Heaviside's own work is not systematically arranged and in places its meaning is rather obscure. Jeffreys also affirms that " · · · it is certain that in a very large class of cases the operational method will give the answer in a page when ordinary methods take five pages, and also that it gives the correct answer when the ordinary methods, through human fallibility, are liable to give a wrong one."

A general idea of the scope of the text may be derived from the chapter headings, which are in order: Fundamental Notions, Complex Theory, Physical Applications: One Independent Variable, Wave Motion in One Dimension, Conduction of Heat in One Dimension, Problems with Spherical or Cylindrical Symmetry, Dispersion, and Bessel Functions. A wealth of illustrative material is compressed within these main divisions. Thus, for example, the third chapter deals with the following topics: (1) variation with time of the electric charge on the condenser plates for a circuit containing a voltaic cell, a condenser, and a coil possessing inductance and resistance; (2) the Wheatstone bridge method of determining inductance; (3) the seismograph, with special reference to the instruments of Galitzin and Milne-Shaw; (4) resonance for a simple pendulum; (5) motion of three particles attached to a stretched string; (6) radioactive disintegration of uranium; and (7) some dynamical applications.

The monograph will be interesting to mathematicians as well as to physicists because it brings out difficulties which require further investigation. For illustration, the first sentence on page 53 is: "A general proof that the results given by the operational method, when applied to the