

THE FIFTY-SIXTH REGULAR MEETING OF THE SAN FRANCISCO SECTION

The fifty-sixth regular meeting of the San Francisco Section was held at the University of California on Saturday, October 27, 1928. The Chairman, Professor E. T. Bell, presided. The total attendance was 37, including the following 23 members of the Society:

Alderton, A. D. B. Andrews, Basoco, E. T. Bell, Bernstein, Buck, Cajori, Corbin, Garver, W. L. Hart, M. W. Haskell, E. R. Hedrick, Hotelling, Frank Irwin, Jerbert, D. N. Lehmer, S. H. Levy, W. A. Manning, W. E. Milne, Noble, T. M. Putnam, A. R. Williams, B. C. Wong.

The following officers were elected for the coming year:

Chairman, Professor M. W. Haskell; Secretary, Professor B. A. Bernstein; Program Committee, Professors E. T. Bell, H. F. Blichfeldt, R. M. Winger, and B. A. Bernstein (ex officio).

It was resolved that the members of the Section living in the Northwest be requested to change the place of the next Summer meeting from the Oregon Agricultural College to the University of California, in order that the Section may participate in the meetings of the Pacific Division of the American Association for the Advancement of Science to be held in June.

It was resolved unanimously: That the Society be requested to make the meetings of the Section Meetings of the Society.

It was decided to hold the next Fall Meeting at the University of California on October 19, 1929.

Titles and abstracts of the papers presented at the meeting follow below. Professor Bell's first paper was given at the request of the Program Committee. The fourth and fifth papers of Professor Bell, Professor Garver's second and third papers, and Professor Whyburn's paper were read by title.

1. Professor E. T. Bell: *The theory of algebraic numbers in the light of Kronecker's program.*

It is suggested as at least plausible, and as a unique piece of scientific irony, that the present unrest concerning the foundations of analysis which was inaugurated by Kronecker half a century ago, reacts destructively upon his own theory of algebraic magnitudes in which it originated.

2. Professor E. T. Bell: *Invariant sequences of polynomials.*

There are determined all sequences of uniform functions of one variable such that the derivative of each term of a given sequence is equal to the preceding term, and each term is changed by a linear transformation of the variable into a multiple of itself, the multiplier being a function of the rank of the term alone, and the linear transformation and the multiplier function being the same for all terms. The terms are necessarily polynomials. By linear transformations on the rank and the variable, any number of distinct sequences having the stated properties for any given linear transformations on their variables, can be replaced by new sequences, all of which are transformed alike by the same linear transformation of the variable, and hence are instances of a single sequence of the original kind. In this respect the theory of any number of distinct sequences of the kind described can be unified. The number of invariant sequences of polynomials is infinite, to the same degree as the set of all uniform functions of one variable.

3. Professor E. T. Bell: *On certain finitely solvable equations between arithmetical functions.*

The number of equations between arithmetical functions for which we can prove either that they have no solution, or only a finite number of solutions, and in the latter case state the solutions, is negligible. Fourteen new equations of this type, involving the binary quadratic class number, are completely solved or are shown to be insolvable. As a point of interest, the solution is achieved by utilizing the conjecture of Descartes in 1638, which was proved only in 1911 by Dubois, (see Dickson's *History*, vol. II, pp. 276, 302).

4. Professor E. T. Bell: *Outline of a theory of arithmetical functions in their algebraic aspects.*

In the Journal of the Indian Mathematical Society for October, 1927, Dr. Vaidyanathswamy calls attention to the existence of inverses of multiplicative arithmetical functions. He remarks that they do not seem to have been utilized before, as they are not explicitly mentioned in Dickson's *History*. The reference in the History which covers them implicitly is vol. I, p. 323, item 174. The present paper is a restatement in more general form of a complete algebraic theory of arithmetical functions, not necessarily multiplicative, which has been constructed by the writer in several papers during the past sixteen years. This restatement generalizes and simplifies all of the preceding work.

5. Professor E. T. Bell: *An interpretation of certain decomposable algebraic forms as functions of divisors.*

The forms are in n indeterminates, where n is an arbitrary constant integer and the coefficients of the n linear factors into which the forms are decomposable are in any given algebraic number field. The decomposition is double, in that it has a dual interpretation, first when the indeterminates are elements of an abstract field, second, when they are elements of the irregular field of all extended numerical functions. When the relevant algebraic number field is defined by a primitive root of unity, the forms degenerate to circulants in each interpretation; another degeneration is that to the resultant of any pair of algebraic equations.

6. Professor Florian Cajori: *Early determinations of the heights of mountains.*

The author makes a detailed examination of the processes and instruments most probably used by the Greeks in determining the heights of mountains. He sets forth the modifications introduced in the Middle Ages and the Renaissance, as well as the improvements in geodetic procedure and in the construction of instruments during the seventeenth and eighteenth centuries. He deals next with the early development of the barometric method, also of the hypsometric and pendulum methods.

7. Professor Florian Cajori: *Hobbes, Wallis, and Barrow on principles of mathematics.*

The author points out that Wallis founded mathematics on arithmetic, that Hobbes and Barrow founded it on geometry. With Wallis arithmetic was an abstract science, to which geometry was subordinate. According to Hobbes, Wallis mistook the study of symbols for the study of geometry. According to Barrow, numbers themselves could not even be added to or subtracted from one another. The discussions carried on involved also the definitions and postulates of Euclid, the consideration of angles of contact and the concept of limits. In particular, Barrow and Wallis defended Euclid's definition of proportion against the attacks of Hobbes who pronounced the definition unworkable because it demanded an infinite number of trials. Hobbes criticises Wallis's reasoning by induction. Wallis anticipated some nineteenth century ideas on arithmetization.

8. Professor Florian Cajori: *Isaac Newton's edition of Varen's geography.*

The author finds that Newton made numerous corrections in Varen's *Geographia Generalis* and added thirty geometric figures. Of particular interest is Newton's correction of a numerical error in Varen's account of the measurement of a degree of the earth's meridian due to Snell, and the use of Snell's value in correcting Varen's table for the distance a mountain of known height can be seen at sea (disregarding refraction). These corrections show conclusively that in 1672 Newton was familiar with the result of Snell's measurement. On no other occasion,

previous to writing the *Principia*, is it known definitely what meridian value Newton actually used.

9. Professor Raymond Garver: *Quartic equations with certain groups.*

A general method, employing the Tschirnhaus transformation, is devised which may be of some value in the problem of determining all equations of a certain degree with a prescribed group. The method can be applied easily to quartic equations; the results are equivalent to those given by Seidelmann in volume 78 of the *Mathematische Annalen*.

10. Professor Raymond Garver: *Linear fractional transformations on quartic equations.*

The reduction of quartic equations to normal forms by means of linear fractional transformations is usually carried out with the aid of either algebraic geometry or invariant theory. This paper considers the problem from a purely algebraic point of view, and treats a number of normal forms.

11. Professor Raymond Garver: *Two notes on cyclic cubics.*

This paper gives (1) a simple proof that every cyclic cubic is of Seidelmann's form $x^3 - 3(p^2 + 3q^2)x + 2p(p^2 + 3q^2) = 0$, and (2) a Tschirnhaus transformation that serves to reduce every such cubic to the normal form $y^3 - 3(P^2 + 3)y + 2P(P^2 + 3) = 0$.

12. Dr. A. R. Jerbert: *Quintuples of curves in four-space.* Preliminary report.

A system of five differential equations of the form $dx_i/dt = a_{ij}x_j$ ($i, j = 1, 2, 3, 4, 5$) serves as analytic basis. A fundamental system of solutions is interpreted as defining, to within a projectivity, 5 points in 4-space. As the parameter t varies, each of the five points describes a curve. The resulting configuration consists, therefore, of five curves in one-to-one correspondence. The projective properties of the configuration are readily expressed in terms of the invariants and covariants of the defining system.

13. Professor W. M. Whyburn: *On the distribution of the characteristic values of differential and integro-differential systems.*

The general system of linear differential equations $\sum_{j=1}^n \sum_{s=0}^k P_{ij}(x)y_j^{(s)} = 0$, ($i = 1, 2, \dots, n$), where the functions ${}_sP_{ij}(x)$ are summable on X : $a \leq x \leq b$, is studied. The matrix calculus is used to develop adjoint relations and the results are shown to coincide with the classical results for the special cases of a single equation of the k th order ($i = j = 1$) and a system of n equations of the first order ($k = 1$). A canonical form is derived for the system when the determinant of the matrix $({}_kP_{ij}(x))$, k fixed, is different from zero on X .

The above differential system is considered when one of the coefficients ${}_rP_{m\ell}$, depends upon a parameter λ in such a way that $P_{m\ell}(x, \lambda)$ is sum-

mable on X for each fixed λ on L : $L_1 < \lambda < L_2$, continuous in λ on L uniformly with respect to x on X , and bounded numerically on XL by a summable function of x . Necessary and sufficient conditions that there exist continua of characteristic values of the differential system and a set of homogeneous boundary conditions are derived. As a corollary of this result it is shown that a self-adjoint system cannot have a continuum of characteristic values and furthermore, the values of λ for which such a system is incompatible form intervals that are everywhere dense on L . Corresponding systems of integro-differential equations are considered.

14. Professor W. A. Manning: *The class of triply and quadruply transitive groups.*

It is known that if a triply transitive group of degree n and class $u (> 3)$ contains a permutation of even order on u letters, $n \leq 2u$; and that for quadruply transitive groups of class $u (> 3)$ in which some permutation of degree u is of even order, $n \leq 2u - 1$. The author now announces that if one of the permutations of degree u of a triply transitive group of class $u (> 3)$ is of odd prime order p , $n < 2pu/(p-1) + 3$. For all quadruply transitive groups of class $u (> 3)$, $n \leq 2u + 1$. This last result betters Bochert's limit by unity.

15. Mr. M. A. Basoco: *On the Fourier expansions of doubly periodic functions of the third kind.*

The doubly periodic functions of the third kind may be classified into two groups according as the excess m of the number of zeros over the number of poles of the function is positive or negative. In a series of papers in the *Annales de l'Ecole Normale*, (1884-1888), Appell has developed a theory for obtaining the Fourier expansion of these functions. It is found that the theory is applicable in a practical manner to functions for which $m < 0$, while for functions such that $m > 0$, the theory, while complete from a function theoretic point of view, does not lead, in general, to arithmetically useful results, since it leaves certain constants expressed in the form of definite integrals, the actual evaluation of which is quite impracticable.

Using other methods, numerous expansions of functions exhibited as quotients of theta products with $m > 0$ have been obtained in a form suitable for use in the theory of numbers.

B. A. BERNSTEIN,
Secretary of the Section.