important matter relating to the development of the theories in Lie's mind. Especially useful in this respect is the account given (pp. 777–793) of the antecedents of memoirs II and III on differential invariants and the differential equations which admit a finite continuous group. This account is mainly in the form of excerpts from letters from Lie to Mayer and Klein. Noteworthy is Lie's insistence (p. 781) on the importance of the synthetic element in the origin of his discoveries. Attention may also be called to the account which Lie gives (pp. 781–782), in a letter to Mayer, of the order in which his memoirs may best be studied.

A free translation of a passage (pp. xxi–xxii) from the editor's introduction will not be without interest:

"If one should go through the whole history of mathematics, I believe that he will not find a second case where, from a few general thoughts, which at first sight do not appear promising, has been developed so extensive and wide-reaching a theory. Considered as an edifice of thought Lie's theory is a work of art which must stir up admiration and astonishment in every mathematician who penetrates it deeply. This work of art appears to me to be a production in every way comparable with that . . . of a Beethoven . . . . It is therefore entirely comprehensible if Lie . . . was embittered that 'deren Wesen, ja Existenz, den Mathematikern fortwährend unbekannt zu sein scheint' (p. 680). This deplorable situation, which Lie himself felt so keenly, exists no longer, at least in Germany. In order to do whatever lies in my power to improve the situation still further, ... I have sought to clarify all the individual matters (Einzelheiten) and all the brief suggestions in these memoirs."

R. D. CARMICHAEL

BORN ON ATOMIC DYNAMICS

Problems of Atomic Dynamics. I. *The Structure of the Atom* (20 lectures). II. *The Lattice Theory of Rigid Bodies* (10 lectures), by Max Born.

(Delivered at the Massachusetts Institute of Technology in 1925–26, and published by the Institute.) 8vo., 200 pp.

The name of the author of this volume is alone sufficient to insure an authoritative presentation of the subjects which it treats. As a synopsis—followed rather closely—of a course of thirty lectures, it has the faults and the merits which such a method of development involves. Each lecture is, to a certain extent, devoted to some special phase of the subject, and this allows the reader to find out easily what the author has to say on any topic. For anyone who is familiar with the general lines which atomic physics has taken during the last ten years this has decided advantages. On the other hand, it produces some feeling of a lack of connectedness. A mathematician who wishes to learn of the developments which have taken place up to a couple of years back is advised to read first some general descriptive account and then to turn to Professor Born's volume. If he has some previous acquaintance with the mathematical methods of which use is made, he should have a sufficient basis to find out what developments the

mathematical side needs in order to assist in the interpretation of the observed phenomena.

The first attempts to explain the structure of the atom were naturally made with the use of the classical theory of dynamics which involved the continuity of space and time and of all phenomena deduced from them. It is true that mass was discontinuous but its motions were always continuous. As observation became more searching, it was found that classical dynamics was no longer sufficient to correlate the phenomena and the idea of the "quantum," or change by finite as opposed to infinitely small steps, was introduced. Certain relations of classical dynamics which involved energy and momentum were retained; others involving rates of change had to be abandoned. Hence Professor Born gives a brief summary of the Hamilton-Jacobi presentation of classical dynamics and follows it with the limitations required by the quantum hypothesis.

Then follow several lectures on Bohr's atomic work which has had such great success in correlating the numerical values of the wave lengths of the spectral lines of the elements and particularly those of hydrogen. The numerous observers, however, soon overtook the mathematicians, and modifications and limitations had to be introduced, even to division of the quantum. The reply was the introduction of the harmonic oscillator and the mathematical theory of matrices and still later, wave mechanics. The physical definition of the first is left conveniently vague—mathematically it is a coordinate which is a simple harmonic function of the time. This has had a certain degree of success and several lectures are devoted to it. But that it is by no means final appears in the last lecture where the application of general observational calculus, introduced by Wien and the author, is briefly outlined. In the same lecture the latter gives his views of the trend of future developments, stressing particularly the necessity for careful consideration of the observed phenomena in the mathematical developments.

The second series is largely devoted to attempts to deduce the various observed properties of rigid bodies and particularly those of crystals, from the theories of atomic structure set forth in the first series. The reading and comprehension of this series constitutes a liberal education in general physics. The general geometrical basis of the lattices is mathematically quite simple; the complications are introduced in the attempts to apply it to various crystals.

The volume is lacking on the personal side. While the names of those who have contributed are freely mentioned throughout, the occurrence of these names appears to be quite incidental. No references are given, and while there is no index, the brief table of contents gives a fairly good idea of the subjects treated. These statements are not made as criticisms but simply as descriptive of the subject matter and manner. There can be no question concerning its value as a summary of the status of the subject at the time of the delivery of the lectures.

E. W. BROWN