

A CORRECTION

BY W. L. AYRES

Dr. G. T. Whyburn has kindly called to my attention the fact that Theorems IV and V of my paper, *Concerning the boundaries of domains of a continuous curve*,* are incorrect. The following example constructed by Whyburn shows that theorem IV is false and a somewhat similar example may be used for Theorem V. Let K denote the rectangle with vertices $(1, 1)$, $(1, -1)$, $(-1, 1)$ and $(-1, -1)$ together with its interior. Let C_{-1} and C_0 be the circles with centers $(0, -1)$ and $(0, 1)$ and radii 1 and $\frac{1}{3}$ respectively. For each positive integer n let C_n be the circle with center $(1/2^n, (-1)^n)$ and radius $5/2^{n+3}$. Let C' and C'' be the set of all points (x, y) of $\sum_{i=-1}^{\infty} C_i$ for which $y > 1$ and $y < -1$ respectively. Let $M = K + C' + C''$. Let D_{-1} be the set of all points (x, y) of K for which $x < 0$ and $y < 0$. Let D_0 be the set of all points (x, y) of K for which $x > \frac{1}{2}$. For each positive integer n let D_n be the set of all points (x, y) of K such that $1/2^{2n} > x > 1/2^{2n+1}$. Let

$$D = C'' + \sum_{i=-1}^{\infty} D_i$$

and let P be the point $(-1, 1)$. The hypothesis of theorem IV is satisfied but one of the maximal connected subsets of the M -boundary of D with respect to P consists of the interval from $(0, 1)$ to $(0, -1)$ together with the interval from $(-1, 0)$ to $(0, 0)$.

I have found that Theorems IV and V are true if *either* of the following conditions be added to the hypotheses:

(4) *If B_1 is any maximal connected subset of B containing more than one point, then B_1 contains two non-cut-points which are accessible from both D and R .* †

* This Bulletin, vol. 33 (1927), pp. 565-571.

† The maximal connected subset of $M - D$ containing P is denoted by R .

(4') If B_1 is any maximal connected subset of B containing more than one point, then the set of non-cut-points of B_1 is accessible from either D or R .

If condition (4) be added, then these theorems may be proved by practically the same argument that I used. If condition (4') be added, these theorems may be established by the following argument. The argument follows my "proof" of Theorem IV through line 7 page 569. As x and y are accessible from either D or R (we will suppose D), there is an arc xuy which lies in D except for x and y . Let T be a maximal connected subset of $B_1 - xzy$. Suppose T has a limit point t on $\langle xzy \rangle$. It is not difficult to see that T contains a non-cut-point s of B_1 . There exists an arc st which lies in T except for the point t .* By (4') there exists an arc rs which lies in D except for s and has only the point r in common with xuy . Now the set $xzy + xuy + rs + st$ divides the plane into three domains, one of which contains R . But if H is the domain containing R there is a point Q of B_1 which belongs neither to H nor to its boundary. Then Q cannot be a limit point of R . Hence every limit point of T belongs to the set $x + y$ and, since neither x nor y is a cut-point of B_1 , both x and y are limit points of T . Then there is an arc xvy which lies in T except for x and y . Since every point of $xvy + xzy$ is a limit point of R , R lies in the complementary domain of $xuy + xvy + xzy$ which has $xvy + xzy$ as its boundary. Then the exterior of $xzy + xvy$ contains either D or R and the interior contains the other. Then $B = xzy + xvy$.

THE UNIVERSITY OF TEXAS

* See R. L. Wilder, *Concerning continuous curves*, *Fundamenta Mathematicae*, vol. 7 (1925), Theorem 1, p. 342.