Analyse Fonctionnelle. By Paul Lévy. Mémorial des Sciences Mathématiques, fasc. V. Paris, Gauthier-Villars, 1925. 56 pp.

The author gives in this brief discussion an exposition of what is necessarily only a part of the subject of functional analysis, but in such a way as to impress the reader with the existence of new ideas under the sun. The condensation naturally enforces slow reading.

Chapter I gives the fundamental notions of function space and the generalized definition of distance, in particular the space Ω, where the square of the distance between two functions is the integral of the square of their difference, and the space ω, in which distance may be defined as the maximum of the numerical value of the difference. The linear functional in the two spaces takes on different forms, in ω being given by a Stieltjes integral and in Ω by a Lebesgue integral; in Ω, orthogonal axes may be regarded as defined by a set of an infinite number of orthogonal functions $f_n(x)$; and a function $\Sigma a_n f_n(x)$ as having coordinates $a_1, \cdots, a_n, \cdots$. Consideration of the vectors in this space yields various trigonometric formulas. Consideration of the linear transformation in the two spaces leads to different types of integral equations.

Chapter II gives an introduction to equations in functional derivatives. The reader will probably turn to references where he can find more detail, but he will be able here to get a notion of complete integrability and of characteristics.

In Chapter III is developed the idea of integration in function space. On account of the fact that the ratio of the volumes of two similar figures becomes infinite or zero as the number of dimensions becomes infinite, the idea of volume must be replaced by that of average. Here again, however, we meet the same difficulty, in that the neighborhood of one position is apt to predominate to an infinite extent in the determination of the average. (This aspect of infinity will be more or less familiar to the reader as the characteristic of one of the well known methods of arriving at Maxwell's law of distribution for the velocities in a gas of a large number of molecules.) It becomes desirable therefore to consider other kinds of weighted means.

The author flirts throughout in engaging fashion with applications to harmonic functions, considering $\partial^2 g(M_1, M_2)/\partial n_1 \partial n_2$ as a function of curves, the equation in functional derivatives of the Dirichlet problem and the nature of harmonic functions in space of an infinite number of dimensions.

The theories expounded owe, of course, to the founders of the subject, and much also to Lévy himself and to Gateaux, Wiener, Fréchet and Daniell.

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