Algebraische Flächen. By H. W. E. Jung. Hannover, Helwigsche Verlagsbuchhandlung, 1925. xvi+410 pp.

This is the logical continuation of the author's recent volume on algebraic curves and is directly in line with the well known treatise of Hensel-Landsberg. It contains in essence his investigations for the last two decades, with a surprising blank at the very outset. The whole edifice rests on the possibility of representing the vicinity of any point on an analytic surface by means of a finite number of sets of power series in two variables. The proofs as now found in the literature (Black, Hensel, and Jung—the best is due to the first named author and is found in PROCEEDINGS OF THE AMERICAN ACADEMY, vol. 37 (1902)) are, however, long and unwieldy, so that the author contents himself with assuming this all-important proposition without proof. This calls for comments that we shall reserve for another occasion.

It will be remembered that the positions on a Riemann surface are treated by Hensel, Landsberg, and Jung as arithmetical divisors. At bottom the associated symbolical operations are in no sense different from those that occur in connection with the Noether-Brill theory of groups of points, elements being merely multiplied instead of added. A similar remark applies to the author's painstakingly defined divisors on an algebraic surface, where things naturally do not go quite so smoothly. His theory, by the way, can be developed with much greater ease by means of some rather simple considerations of a topological nature. Be that as it may Jung is mainly interested in the projective group; hence his book, which concerns itself essentially with certain intersection numbers and projective invariants, is much more in line with the school of Cayley and Sylvester than with the work of modern algebraic geometers. Of the invariant integers which play such an important part in their work that of Zeuthen-Segre alone receives much attention from him. In a treatise published at the present time we would have enjoyed seeing a more varied selection of topics. However, within the narrow scope deliberately chosen by the author, he has done well indeed and geometers will read him with interest and profit.

S. LEFSCHETZ

Die mathematischen Hilfsmittel des Physikers. By Erwin Madelung. Second edition. Berlin, Julius Springer, 1925. xiii+283 pp.

This monograph constitutes the fourth volume of the collection edited by R. Courant and entitled DIE GRUNDLEHREN DER MATHEMATISCHEN WISSENSCHAFTEN IN EINZELDARSTELLUNG. It contains statements of all the fundamental principles and outlines of all the methods which, in the author's opinion, may be required by the theoretical or computing physicist. In the chapters which deal with pure mathematics practically everything is included that has been used heretofore in the solution of physical problems, as well as additional material that gives promise of future applicability. Of course, the territory of the common tables of integrals, numerical data, etc., is not encroached upon.

Proofs are indicated only in a few places. With these exceptions, the presentation is consistently limited to definitions of the fundamental concepts, to the associated formulations, and to the final results. Fortunately especial stress is laid on the algebra of vectors and tensors.

The first ten chapters are devoted to pure mathematics as may be inferred from the following chief headings: algebra, differential and integral calculus, series, functions, transformations, differential equations, linear integral equations, calculus of variations, theory of probability, and vector analysis. The remaining chapters and the appendix really constitute an outline of a valuable course in theoretical physics. More specificially, the fields represented are mechanics, electricity, theory of relativity, thermodynamics, quantum theory, and a few physical applications of the calculus of probability. A short list of authoritative references is appended to each chapter.

The second edition differs from the first principally in certain additions such as boundary problems, tensor analysis, theory of the top, the general relativity theory, and the quantum theory. The printing is very clear, the spacing is adequate, and typographical errors seem to be absent. In the reviewer's opinion this book merits the attention of all lecturers on applied mathematics and a working knowledge of its contents should be a minimum requirement in mathematics of all candidates for the degree of doctor of philosophy in physics.

H. S. UHLER

Binomial Factorizations, Vol. V. By A. J. C. Cunningham. London, Francis Hodgson, 1924. lxxii+120 pp.

This volume is one of a set of seven giving the smallest roots of the congruence $y^n \pm 1 \equiv 0 \pmod{p}$ with p running through the available primes (of the form $2nx+1$) to various high limits. From these tables the author builds up factorization tables for different values of y. Thus from the table of roots of $y^{11}-1\equiv 0 \pmod{89}$, namely: 2, 4, 8, 16, 32, 39, 45, 64, 67, 78, he is able to insert a factor 89 in the list of divisors of what he calls "Simple Undecimans" (that is to say in the divisors of $(y^{11}-1)/(y-1)$) opposite the corresponding values of y. This table of "Simple Undecimans" runs to $y=100$, and contains only 36 complete factorizations. To get these factorizations over nine hundred congruences had to be solved and over nine thousand roots listed. The corresponding table $n=13$ gives only 26 complete factorizations in the first hundred "Simple Tredecimans" as the result of solving some 800 congruences and of listing some 9,600 roots. The showing for "Simple Septimans" ($n=7$) given in previous volumes is not much better. However, although the percentage of complete factorizations is not impressively large, nevertheless the incomplete factorization of these high numbers will serve to settle the question of their primality and often this is all that is desired. Moreover the determination of one factor will make the complete factorization by other methods easier to perform.

D. N. LEHMER