SHORTER NOTICES

Elementi del Calcolo delle Variazioni. By G. Vivanti. Bibliotheca di matematiche superiori diretta da Roberto Marcolongo e Gaetano Scorza. Messina, Giuseppe Principato, 1923. 296 pp.

One of the most interesting features of this book is the large number of examples in the calculus of variations which are discussed throughout the text. Theoretical questions have not been neglected, but after treating them concisely the author hastens to elucidate his theory by applying it to numerous illustrations. He has followed the example set by Kneser in listing his problems by number at the end of the book, with indications of the pages on which they are considered. There are twenty-four in all. Most of them are well known, but one may be especially mentioned. It is an isoperimetric problem (No. 14, p. 93) whose extremals are straight lines, which has only solutions with corners, and whose conjugate points are determined by a simple geometric construction.

Mathematicians value highly the books on algebraic geometry which are devoted to the theory of special algebraic curves or surfaces. It has always seemed to me that for similar reasons a book devoted entirely to the elucidation of special problems of the calculus of variations, in which the theory of each problem would be developed consecutively as far as it is known, would be of great value. The text of Professor Vivanti would give important assistance in preparing such a treatise.

The book before us is divided into two principal parts, devoted, respectively, to the conditions arising from the first and second variations, and there is an appendix on functional calculus and its applications to the proof of the existence of absolute minima. The variety of problems of the calculus of variations treated is large, the more important ones being the simplest problem in the plane, parametric problems, problems whose solutions have corners or arcs in common with the boundary of a region, isoperimetric problems, problems with variable end-points, problems of the Lagrange type, and those involving double integrals. One chapter in each part is devoted to each of these cases. The simplest problem in the plane in non-parametric and parametric forms is treated with considerable completeness, but the conditions deduced for the other problems are for the most part necessary conditions. In a number of cases references are given to Bolza for sufficiency proofs, and in others the proofs are omitted or passed over somewhat hastily.

I should like to call attention here to the method which I have used for deducing the necessary condition of Jacobi from the second variation.* It is widely applicable and relatively simple. In parametric cases particularly

* TRANSACTIONS OF THIS SOCIETY, vol. 17 (1916), p. 195; Bliss, *Calculus of Variations*, p. 161; Smith, TRANSACTIONS OF THIS SOCIETY, vol. 17 (1916), p. 459; Larew, *ibid.*, vol. 20 (1919), p. 1.

it avoids the use of the transformation of Weierstrass which is exceedingly ingenious and interesting, but complicated and not easy to generalize for spaces of higher dimensions. The method used by Vivanti is the historically very important one which has come to us through a process of development from the ideas in Legendre's fundamental memoir on the second variation.

It seems to me that the theoretical material in this interesting book on the calculus of variations is presented in a somewhat less clear and systematic style than that which one finds in the other very important and useful treatises which we have had from the pen of Professor Vivanti. This is doubtless due to the author's justifiable desire to arrive as rapidly as possible at the results which are of significance for the interesting special problems which he considers. His conciseness has led to some inaccuracies, as in the reference on page 132 to Bolza for a part of the proof of the Euler-Lagranbe multiplier rule. The text seems to imply a theorem which Bolza has not proved and which I do not believe to be correct. Limitations of space probably forbade the expansion of the material in the appendix. It is at present a very interesting outline of a chapter in the calculus of variations which is still in process of development. Professor Vivanti there regards the integrals of the calculus of variations as special instances of functions of lines, and applies to them Volterra's definition of the derivative of such a function. In order to be effective for these integrals, however, the definition must be modified slightly. To secure the derivative limit, one must make the first and second derivatives of the variation $\eta(x)$ approach zero, as well as impose the Volterra requirement that the variation itself and the interval on which the variation is different from zero are infinitesimals.* I believe that the differential corresponding to Volterra's derivative, or the differential of Fréchet, for a function of a line, would prove to be more convenient than the derivative in this connection. The definition of a generalized integral on page 289 needs modification if it is to be applicable to the non-parametric integrals which are considered in the preceding paragraphs.

G. A. BLISS

Maschinenbauliche Beispiele für Konstruktionsübungen zur darstellenden Geometrie. By Theodor Schmid. Leipzig and Vienna, Franz Deuticke, 1925. 52 pp., 25 plates.

This is a collection of a number of very simple examples in mechanical drawing for the use of the young "technician," who "as a rule is not able to apply the instruction which he got in the usual courses in descriptive geometry to practical examples."

To judge from the practical examples represented on the 25 plates, the courses in descriptive geometry which do not enable a student to apply the knowledge gained to the representation of such simple objects must indeed be in a bad state. In the opinion of the reviewer very little descriptive geometry is needed to understand such plates.

ARNOLD EMCH

* Proceedings of the NATIONAL ACADEMY OF SCIENCE, vol. 1 (1915), p. 173.