THE MAY MEETING IN NEW YORK

The two hundred forty-ninth regular meeting of the Society was held at Columbia University, on Saturday, May 1, 1926, extending through the usual morning and afternoon sessions. The attendance included the following sixty-four members:

W. L. Ayres, C. R. Ballantine, J. P. Ballantine, A. A. Bennett, Eisenhart, Fiske, Fite, Fort, Frink, Gafafer, Garretson, Gill, R. L. Green, Gronwall, Hansell, Hedlund, Hille, Huntington, Joffe, Kasner, Kormes, Lamond, Lamson, Langford, Langman, Littauer, Maria, Meyer, H. H. Mitchell, L. T. Moore, T. W. Moore, Murnaghan, Norwood, B. C. Patterson, Pfeiffer, Pierpont, R. G. Putnam, Rainich, Raudenbush, Raynor, Reddick, Ritt, Schwartz, Seely, Siceloff, M. H. Stone, J. H. Taylor, Teach, J. M. Thomas, Tracey, Veblen, H. E. Webb, Weida, Weiss, Pell-Wheeler, Whittemore, W. A. Wilson.

There was no meeting of the Council or of the Trustees.

Professor E. W. Brown presided at the morning session, relieved by Professor Oswald Veblen; in the afternoon Professor L. P. Eisenhart presided, relieved by Professor J. F. Ritt.

At the request of the Program Committee, Professor H. H. Mitchell delivered an address, at the beginning of the afternoon session, entitled *Asymptotic laws in the theory of numbers*. Titles and abstracts of the other papers read at the meeting follow below. The papers of Adams, Carman, Cleveland, Emch, Foster, Gehman, Georges, Glenn, Graves, Gronwall (second paper), Hille, Hollcroft, Kellogg, Levy, Lubben, Maria, Morley, Robertson, Robinson, Tamarkin, Tamarkin and Langer, Vandiver, and Whyburn were read by title. Mr. Cleveland and Mr. Whyburn were introduced by Professor R. L. Moore, Mr. Georges by Professor Dickson, Professor LaMer by Dr. Gronwall, and Professor Starke by Professor Ritt.

1. Mr. T. W. Moore: *Notes on the rational plane cubic curve*.

This paper appeared in full in the May-June issue of this BULLETIN.

2. Professor T. R. Hollcroft: *On harmonic cubics.*

It has long been known that the Hessian of the Hessian of a harmonic cubic is the original cubic. The process of obtaining the Hessian of a canonical cubic may be considered merely a repeated transformation of the parameter. Define the process of obtaining the Cayleyan of a canonical cubic successively as a similar repeated transformation of the parameter, the curve being dualized each time the transformation is applied. Then it is also true that the Cayleyan of the Cayleyan of a harmonic cubic is the cubic itself. Moreover, the Cayleyan of any harmonic cubic is harmonic and when the cubic is real, the Cayleyan has the same parameter as the cubic. The condition that the parameter of the Cayleyan of the Hessian equal the parameter of the Hessian of the dualized Cayleyan of a given cubic is satisfied by all the fourteen special cubics of the syzygetic pencil, six harmonic, four equianharmonic, and four degenerate, and by no others.

3. Professor James Pierpont: *Note on the horospheres.*

This paper will appear in an early issue of this BULLETIN.

4. Professor Frank Morley: *Note on differential inversive geometry.*

In the memoirs of Mullins,* Liebmann,† and Kubota‡ the differential inversive geometry of a planar curve is firmly established. The following way of approach seems desirable. Denote the Schwarzian derivatives of x as to y by $\{x,y\}$. We have, for any number of connected variables Cayley's cyclic formula (say C_n), $\{x,y\}(dy)^2+\{y,z\}(dz)^2+\cdots+\{l,x\}(dx)^2=0$. Let the curve in question be given by a self-conjugate equation between x and its conjugate $\bar{x}$. Then by C_2, $\{x,\bar{x}\}(d)\bar{x}^2+\{\bar{x},x\}(dx)^2=0$; so that $\{x,\}\bar{x}(d\bar{x})^2=2i(dt)^2$, where t is real. Let us next apply a homography $x=(ay+b)/(cy+d)$, and use C_4 for the cycle $x,\bar{x},\bar{y},y$. Since $\{y,x\}=0$ and $\{\bar{x},\bar{y}\}=0$, we have $\{x,\bar{x}\}(d\bar{x})^2+\{\bar{y},y\}(dy)^2=0$, or $\{x,\bar{x}\{(d\bar{x})^2=\{y,\bar{y}\}(dy)^2$. Hence t is an invariant under homographies, and is the proper real parameter. Using C_3, we find $\{x,t\}(dt)^2-\{\bar{x},t\}(dt)^2=\{x,\bar{x}\{(dx)^2=2i(dt)^2$; so that $\{x,t\}=i+I$, I being the fundamental differential invariant.

5. Mr. B. C. Patterson: *A formula for the inversive curvature.*

If a curve is given as a complex function of a real parameter, let the Schwarzian derivative of this function be separated into its real and imaginary parts, A and iB. The inversive curvature is then given by the formula $I_5=(8AB^2-4BB''+5B'^2)/8B^3$, where the primes indicate differentiation with respect to the parameter.

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* *Differential Invariants under the Inversion Group*, Columbia dissertation, Lancaster, Pa., 1917.

† *Beiträge zur Inversionsgeometrie der Kurven*, MÜNCHENER BERICHTE, 1923.

‡ *Beiträge zur Inversionsgeometrie*, SCIENCE REPORTS, TÔHOKU UNIVERSITY, Mathematical Section, vol. 3 (1924), No. 1.

6. Mr. G. Y. Rainich: *Remarks on differential inversive geometry.*

(1) If we call inversive distance between two linear elements of the second order the square root of the sine of the angle (which may be imaginary) formed by the two circles determined by the elements, the inversive length of arc may be defined as the limit toward which tends the sum of inversive distances between consecutive elements on the arc when these distances tend towards zero. (2) It is known that using dual numbers the Laguerre inversions may be given in the form of fractional linear transformations. With the aid of this representation, two seminvariants of plane curves under the Laguerre group are obtained, by forming the Schwarzian derivative, and from these the differential and integral invariants are derived, in the same way as this is done for point inversions.

7. Mr. G. Y. Rainich: *A type of space-time.*

The expression in vector form is given for a Riemann tensor which allows a two-parameter group of rotations. This tensor depends on eight constants. The Bianchi equations are written out in vector form, and a type of space-time the Riemann tensor of which depends on seven constants is discussed, for which the radiation vector q vanishes in regions of regularity, and, as a consequence, Maxwell's equations are satisfied.

8. Professor F. D. Murnaghan: *A contribution to Maxwell's theory of dielectric absorption.*

According to Maxwell, the residual or "anomalous" currents which follow the charge or discharge of a dielectric are due to a lack of homogeneity of the dielectric, and he indicated a theory of a layer dielectric but did not carry out the solution. The current is given by a linear integral equation of Volterra's type whose solution consists of a finite number of exponential terms. The complete solution for continuous and alternating E.M.F.'s follows readily. The solution furnishes a proof of the important Boltzmann-Hopkinson empirical "principle of superposition."

9. Mr. W. L. Ayres: *Continuous curves of a certain type.*

S. Mazurkiewicz,* R. L. Wilder,† and others have considered continuous curves which contain no simple closed curve. The present paper considers continuous curves containing only a finite number of simple closed curves, and it is found that these curves possess many properties similar to those proved for the above type. Among other properties given, it is shown that for this type every connected subset is arc-wise connected, every closed connected subset is a continuous curve, and every boundary point of an S-domain of such a curve is accessible. Some of the properties

* *Un théorème sur les lignes de Jordan*, FUNDAMENTA MATHEMATICAE, vol. 2 (1921), pp. 119–30.

† *Concerning continuous curves*, FUNDAMENTA MATHEMATICAE, vol. 7 (1925), pp. 341–77.

are possessed also by the more general type such that, given any positive number ϵ, the continuous curve contains only a finite number of simple closed curves of diameter greater than ϵ.

10. Mr. W. L. Ayres: *On continuous* (1,1) *correspondences between continuous curves.*

The author considers the following problem. Suppose M and M' are continuous curves and there exists a (1,1) correspondence between K and K', subsets of M and M' respectively. Under what conditions can this correspondence be extended to a (1,1) continuous correspondence between the curves themselves? In this paper the conditions are given for the cases where the continuous curves contain no simple closed curve and where they contain just one simple closed curve. Branch points are defined, and it is proved that in the two cases under consideration the set of branch points is countable. The conditions given are conditions on the branch points of the two curves and on K and K'. For the case where K and K' are vacuous, this gives a set of conditions on the branch points under which two continuous curves can be put into (1,1) continuous correspondence. The conditions given are independently necessary.

11. Mr. Mark Kormes: *On arithmetic summation of point sets.*

Arithmetic summation of point sets is defined as summation of the coordinates of their points. The following theorems are shown to hold: (1) the arithmetic sum of two sets having positive measure contains at least one interval; (2) the arithmetic sum of a single set of positive measure contains at least one interval; (3) the preceding theorems are true if we replace the sets having positive measure by sets the complements of which with respect to an interval are of the first category.

12. Dr. C. H. Langford: *On two new quantifiers for general propositions.*

General propositions are commonly constructed in terms of the two applicatives "some" and "every." These applicatives occur in functions of the form $(\exists x) \cdot \phi x$ and $(x) \cdot \phi x$. There are four quantitative functions of a property $\phi\hat{x}$, namely, every x is such that ϕx, some x is such that ϕx there is no x such that ϕx, and not every x is such that ϕx. "No x is such that ϕx" is denoted by $[x] \cdot \phi x$, and "not every x is such that ϕx" by $\{x\} \cdot \phi x$. In connection with multiply-quantified functions both of these latter applicatives have formal properties which are in some respects simpler than the properties of "some" and "every." $[x] \cdot \phi x$ generalizes Sheffer's function p "stroke" q when interpreted "p and q are both false" and $\{x\} \cdot \phi x$ generalizes p "stroke" q when interpreted "p is false or q is false."

13. Dr. M. G. Carman: *Expansion problems in connection with homogeneous linear q-difference equations.*

Expansions of the Neumann and Gegenbauer types are obtained for analytic functions in terms of solutions of the general homogeneous linear q-difference equation with analytic coefficients. Expansions analogous to ascending power series, descending power series, and Laurent series are derived. The question of expanding functions of several variables is considered. This paper will appear in an early issue of the TRANSACTIONS OF THIS SOCIETY.

14. Dr. Rachel B. Adams: *The solutions of Fredholm's homogeneous integral equation as functionals of the kernel.*

This paper deals with the Fredholm homogeneous integral equation of the second kind in which the kernel $K(x,y)$ is real and continuous in the region S considered. We first show that the characteristic numbers vary continuously with the kernel. Let, then, λ_0 of multiplicity n and $\bar{\lambda}_i$ $(i=1, 2, \cdots, n)$ be corresponding characteristic numbers for $K(x,y)$ and $\bar{K}(x,y)$ respectively. If the indices of λ_0 and of the $\bar{\lambda}_i$ are equal [unequal] to the corresponding multiplicities, it is shown that given an arbitrarily small positive quantity ϵ, a positive number δ can be found such that if $|K(x,y)-\bar{K}(x,y)|$ is $<\delta$ throughout S, the difference between each of the n normalized fundamental solutions (Lalesco) [each of the normalized fundamental solutions] of either kernel and a suitable linear combination of the n normalized fundamental solutions [functions] of the other is in absolute value less than ϵ. It is of interest to note that we thus obtain for both cases approximate solutions of the given integral equation. A method is developed for determining the closeness of this approximation.

15. Professor J. Tamarkin: *On Laplace's integral equation.*

This paper deals with the solution $F(x)$ of the equation

$$\int_0^\infty e^{-2x\xi} F(\xi)d\xi = f(z),$$

as a function of the complex variable x. Certain properties of reciprocity between $f(z)$ and $F(x)$ are established.

16. Professor J. Tamarkin: *On Volterra's integro-functional equation.*

This paper is devoted to the discussion of the theorems of existence and uniqueness of solutions of the integro-functional equation

$$u(x) = f(x) + s(x)u[\theta_1(x)] + \int_{-x}^{x} K(x,\xi)u[\theta_2(\xi)]d\xi,$$

where $u(x)$ is the unknown function and all other functions are given.

17. Professors J. Tamarkin and R. E. Langer: *A notion of uniform integrability.*

This paper appears in full in the present issue of this BULLETIN.

18. Professor H. S. Vandiver: *On the least multiple of an integer expressible as a definite quadratic form.*

This paper contains a proof of the following theorem: If $r^2+a\equiv 0$

(mod n), where $a>0$, a is not a square and prime to $n>1$, then there exists at least one set (x,y) such that $x^2+ay^2=kn$, $k\leqq 2\sqrt{a/3}$, and $ry\equiv \pm x$ (mod n), x and y being integers. The proof is easily obtained from the theory of definite quadratic forms. The result is required in connection with one of the author's methods of factoring large integers.

19. Professor H. S. Vandiver: *Summary of results and proofs concerning Fermat's last theorem.* Second note.

In this note, among other results, the author indicates the proofs of the following theorems: If p is an odd prime, and $u^p+v^p+w^p=0$ is satisfied in non-zero integers in the field $\Omega(\alpha+\alpha^{-1})$, $\alpha=e^{2i\pi/p}$, then the class number of the field $\Omega(\alpha)$ is divisible by p^2. Under the assumptions (1) none of the Bernoulli numbers B_k, $k=(sp+1)/2$, $B_1=1/6$, $B_2=1/30$, etc. ($s=1, 3, \cdots, p-4$) are divisible by p^2, for p an odd prime, and (2) the second factor of the class number of the field $\Omega(\alpha)$ is prime to p, the equation $x^p+y^p+z^p=0$ is not solvable in rational integers, none zero.

20. Dr. H. M. Gehman (National Research Fellow): *On extending a continuous (1,1) correspondence of two closed point sets.*

In this paper it is proved that if M and M' are closed point sets lying in euclidean n-spaces L and L' respectively, and if S and S' are euclidean $(n+1)$-spaces containing L and L' respectively, and if there exists a continuous (1,1) correspondence T, such that $T(M)=M'$, then there exists a continuous (1,1) correspondence U, such that $U(S)=S'$, and such that for any point P of M, $U(P)=T(P)$. This theorem has already been proved for the case $n=1$ by R. L. Moore (AMERICAN JOURNAL, vol. 48 (1926), p. 67).

21. Dr. R. G. Lubben: *Concerning fixed points in an acyclic continuum undergoing a continuous transformation.*

If K is a bounded continuum such that no sub-continuum of K other than a point is its own prime part, not more than one sub-continuum of K is irreducible between any two given points of K, and T is a continuous one-to-one transformation, with a single-valued inverse, of K into itself, then T leaves fixed at least one point of K.

22. Mr. G. T. Whyburn: *Concerning domains and their boundaries.*

In this paper the following theorems are established. (I) In order that a continuous curve M should be the boundary of a connected domain it is necessary and sufficient that if J denotes any simple closed curve belonging to M, I its interior, and E its exterior, then (a) M is a subset either of $J+I$ or of $J+E$, and (b) if A and B denote any two points of J, then $M-(A+B)$ is not connected. (II) If the boundary M of a domain D is a continuous curve, and if K denotes the set of all the cut points of M, then

$D+K$ is uniformly connected im kleinen. (III) If the boundary M of a simply connected bounded domain D has only a finite number of complementary domains, then in order that M should be a continuous curve it is necessary and sufficient that M should contain a closed point set K such that (a) $D+K$ is uniformly connected im kleinen, (b) every maximal connected subset of K is a continuous curve, and (c) for every number $\epsilon>0$, there are only a finite number of maximal connected subsets of K of diameter $>\epsilon$.

23. Mr. G. T. Whyburn: *Two-way continuous curves.*

A continuous curve M will be said to be "two-way continuous" if and only if between every two points of M there exists in M at least two arcs neither of which is a subset of the other. The following theorems are established. (1) A continuous curve M is two-way continuous if and only if every arc of M contains a non-cut point of M. (2) A continuous curve M is two-way continuous if and only if the set of all the cut points of M contains no continuum. (3) The boundary of a complementary domain of a two-way continuous curve is itself two-way continuous. (4) The boundary M of a complementary domain of a continuous curve is two-way continuous if and only if M contains a point set K such that (a) $D+K$ is uniformly connected im kleinen, and (b) every arc, if any, which $\bar{K}$ (K plus its limit points) contains, contains a non-cut point of M. (5) If M is any continuum whatever, and if L denotes the set of all the cut points of M, then every continuum which L contains is a continuous curve.

24. Mr. C. M. Cleveland: *Conditions under which two points of a continuous curve are separated by a simple closed curve lying therein.*

The author establishes the following theorem. If A and B are two distinct points of a continuous curve M such that any simple continuous arc from A to B contains at least one point of M distinct from A and from B, then there exists a simple closed curve lying wholly in M and separating A from B.

25. Dr. L. M. Graves (National Research Fellow): *Implicit functions and differential equations in general analysis.*

The chief purpose of this paper is to discuss some special cases of the implicit function theorems obtained by Hildebrandt and Graves in a paper entitled *Implicit functions and their differentials in general analysis.* A number of general theorems are first given, which facilitate application of the general theory to special cases. An "extended" implicit function theorem is obtained by a new method from the original implicit function theorem. In the treatment of existence and imbedding theorems for ordinary "differential equations," continuity restrictions are removed and no mention is made of derivatives. The treatment seems to be the most general that is possible with use of a Lipschitz condition. As a special case, imbedding and existence theorems for solutions of equations in the form

$f(x, \int_a^x g(x, x', y(x'), c)dx', y(x), c) = 0$ are treated in detail. The functions f, g, and y are supposed to be bounded and measurable in the single real variable x. Finally, some special properties of solutions of linear "differential" equations are discussed.

26. Dr. H. P. Robertson (National Research Fellow): *Manifolds with non-Riemannian line element which admit geodesic coordinates.*

It is asked under what conditions an n-dimensional manifold, whose metrical properties are determined by an invariant quartic differential form, admits geodesic coordinates. This requires that the $(1/24)n(n+1)(n+2)(n+3)$ coefficients of the form are subject to $(1/24)n^2(n^2-1)(n+6)$ first order differential equations linear and homogeneous in the derivatives. These equations have been solved for the case $n=2$, the case treated by the classical differential geometry, and show that the only surfaces for which the quartic does not degenerate into the square of a quadratic are those in which the line element may be reduced on transformation to one with constant coefficients. The analogous solutions satisfy the equations for any value of n, but for $n>2$ the above method becomes too cumbersome, so the search for other possible solutions is being continued by the methods of the absolute differential calculus. Of particular interest from the physical standpoint is the case $n=4$, as its solution will give the only possible "physical universes" which are in harmony with the postulates of the theory of relativity concerning the nature of force.

27. Dr. Harry Levy (National Research Fellow): *The canonical form of the ds^2 of spaces for which the five-index symbols of Riemann vanish.*

The author shows that the ds^2 of spaces for which the five-index symbols of Riemann vanish is expressible as the sum of forms, $ds^2 = \sum d\sigma_\alpha^2$, where $d\sigma_\alpha^2$ is a form of constant curvature and such that the variables of any one $d\sigma_\alpha^2$ appear only in that $d\sigma_\alpha^2$.

28. Dr. Harry Levy: *Some properties of spaces for which the five-index symbols of Riemann vanish.*

Application is made of the results of the preceding paper to obtain several properties of those spaces for which the five-index symbols of Riemann vanish. In particular, the linear element of the most general space which can be mapped geodesically upon a given space of the above type is found to be $ds^2 = \sum_\alpha \rho_\alpha d\sigma_\alpha^2$, where the ρ_α are constants, a priori arbitrary, and where $ds^2 = \sum_1 d\sigma_\alpha^2$ is the ds^2 of the given space. Other properties are obtained; also conformal maps between two spaces of this type are studied.

29. Dr. Harry Levy: *Congruences of curves in the geometry of paths.*

The author considers, in the geometry of paths, n independent congruences of curves, and, upon them as basis, he develops a theory which is quite analogous to the theory of the orthogonal ennuple in a Riemann space as developed by Ricci, and which includes this theory due to Ricci as a special case. Certain functions of the ennuple are obtained and then found to be invariant under transformations of coordinates and to have properties which mark them as being the analogues of Ricci's coefficients of rotation. Properties of congruences of curves or of the space itself are expressed in terms of these invariants. In addition *projective* invariants are discussed, and in terms of them projective properties of the space are expressed.

30. Professor M. C. Foster: *Ruled surfaces referred to the trihedral of a directrix.*

The directrix is taken as any curved line C upon the ruled surface. The displacements of any point on the ruling through a point M of C are considered relative to the moving trihedral of C at M. Many of the well known theorems concerning ruled surfaces are proved by this method. The paper contains certain theorems relative to ruled surfaces for which the line of striction is an asymptotic line, and to the locus of the center of geodesic curvature of an orthogonal trajectory of the rulings.

31. Dr. A. J. Maria (National Research Fellow): *Stieltjes derivatives.*

In this paper we consider the derivative of a completely additive function of sets $f(e)$ with respect to $g(e)$ ($g(e)$ a completely additive function of sets and not necessarily the measure function). Theorems are obtained concerning the decomposition of $f(e)$ into a singular function and a function which has $g(e)$ as a basis function.

32. Professor Arnold Emch: *On the discriminant of ternary forms and a certain class of surfaces.*

In a paper entitled *On the Weddle surface and some analogous loci,** the author studied the class of surfaces and curves defined as the loci of vertices of general cones passing through certain definite numbers of generic fixed points in space. This investigation may be extended to such systems of cones with nodal generatrices. For this purpose, it is obviously necessary to make use of the properties of the discriminant of a ternary form, to which the intersection of a cone with a generic plane reduces. The principle result is contained in the following theorem: The locus of vertices of n-ic cones through $(1/2)n(n+3)$ generic points in space is a surface of order

* TRANSACTIONS OF THIS SOCIETY, vol. 27 (1925), pp. 270–78.

$(1/2)(n-1)^2n^2(n+3)$, which has each of these points as a multiple point of order $3n(n-1)^2$, and each join of two of these points as a multiple line of order $3(n-1)^2$. As an example, the locus of vertices of cubic cones is investigated in detail.

33. Mr. J. S. Georges: *On division algebras corresponding to abelian groups with n abelian generators.*

This paper is a generalization of the results obtained by L. E. Dickson on an independent set of associativity conditions for division algebras corresponding to an abelian group with two independent generators, and consists of a complete determination of an independent set of associativity conditions, between the parameters, for division algebras corresponding to abelian groups having a set of n independent generators.

34. Professor Einar Hille: *Applications of a Tauberian theorem to the convergence theory of orthogonal series.* Preliminary report.

Littlewood has proved that a series is convergent to the sum A, if (*i*) it is summable Abel to A, and (*ii*) the terms are $O(1/n)$. As far as the author knows, this theorem has not previously been applied to the convergence theory of orthogonal series other than Fourier series. In applying the theorem to other series we have to prove summability Abel of $\sum f_n u_n(x)$ for certain functions $f(x)$, and to give examples of functions for which $f_n u_n(x) = O(1/n)$. Theorems on summability Abel are known in a few important cases, and can be proved for more extensive classes of expansions. One can frequently prove that the terms are $O(1/n)$ when $f(x)$ is a saltus function; it follows that certain classes of functions which can be approximated by saltus functions have the same property. If $f(x)$ is of bounded variation and there exists a $p(x)$ such that $c_n = \int_a^b p(x)u_n(x)df(x)$ can be expressed linearly in terms of the f_n, then it is sometimes possible to obtain such an estimate of c_n that the terms in the orthogonal series become $O(1/n)$. The author has tried out these ideas with a good deal of success on certain expansions in terms of polynomials (Hermite's, Laguerre's, and Legendre's) or boundary value functions of the Sturm-Liouville type.

35. Dr. Tobias Dantzig: *On systems of conics associated with point-to-point transformations of the plane.*

Let T be a continuous point-to-point transformation of the plane in itself, X and Y two corresponding points. The line elements through X and Y are in homographic correspondence, and the intersections of corresponding rays determine a conic (C) through X and Y. The aggregate of the conics (C) form a two-parameter family (S), the indicating system of the transformation T. This system was considered in an earlier paper by the author (American Journal, 1919). In the present paper the inverse problem is discussed: Given a two-parameter family of conics (S), to determine the transformations T which admit S for an indicating system.

For the general case, the conditions for the existence of T are determined in the form of partial differential equations connecting the canonical parameters of the conic (C) with the two parameters of the system (S). Results of geometric interest are obtained for certain special cases.

36. Mr. P. D. Schwartz: *On the classification of conics in hyperbolic geometry.*

In this paper a third degree equation is derived whose roots determine the poles of an ortho-polar triangle. By considering the different combinations of the roots of the cubic, and by taking first poles not on the absolute and second poles on the absolute, and by means of transformations, the conics in hyperbolic geometry are classified.

37. Professor O. E. Glenn: *On biternary transvection and reducing series with application to systems of invariants of connexes.* Preliminary report.

Although several mathematicians have proposed forms of generalization of Gordan's series, the theory, as discovered to date, leaves much to be desired. The present writer has identified a new algorithm in the symbolic theory which extends the most general form of the reducing series of Gordan to the case of biternary concomitants. This sequence, as applied to the problem of reduction of ternariants, approximates in utility to the binary series of Stroh, for which the claim was made that it would give all reductions. Applications in this paper in its present form include a new derivation of the known system of eight concomitants of $a_x \alpha_u$ (Clebsch and Gordan) and the fundamental forms of $a_x^2 \alpha_u$ of the lower degrees. The number of invariant formations for $a_x^m \alpha_u^n$ increases with such rapidity with the orders (m,n) that a theory of symbolic moduli is desirable and is being developed

38. Professor E. P. Starke: *Certain uniform functions of rational functions.*

The functions $\cos z$ and $\wp(z)$, being even, are uniform functions of z^2. This suggests examining various classes of meromorphic functions, to determine which ones among them have the property of being uniform functions of non-linear rational functions. In this paper, the examination is carried out for the periodic functions and for the Poincaré functions (with rational multiplication theorems). We show that periodic functions with the above property either are meromorphic functions of $\cos(\lambda z+\mu)$, or are rational functions of $\wp(z+\mu)$, or of $\wp^2(z+\mu)$ if $g_3=0$, or of $\wp'(z+\mu)$ or of $\wp^3[(z+\mu)$ if $g_2=0$. In regard to the Poincaré functions, we note that if in one such function, we replace z by any integral power of z, we have another Poincaré function with the property we seek. All Poincaré functions which satisfy our requirements and cannot be obtained in the above manner are proved periodic. They are then linear functions of $\cos(\lambda z+\mu)$, or of $\wp(z+\mu)$, or of $\wp^2(z+\mu)$ if $g_3=0$, or of $\wp^1(z+\mu)$ or $\wp^3[(z+\mu)$ if $g_2=0$, with restrictions on the values of μ.

39. Dr. T. H. Gronwall and Professor V. K. LaMer: *Internal and free energy in their relation to the Debye-Hückel theory.*

It is shown that, contrary to previous statements in the literature, the validity of the Debye and Hückel derivation of the expression for the free energy from that of the internal energy is in no wise dependent on the assumption (which is contrary to experimental facts) that the dielectric constant is independent of the temperature. Conditions on the dielectric constant, sufficient for the validity of the theory, are given, and these conditions do not disagree with any of the experimental data. The remainder of the paper discusses certain thermodynamical errors which have appeared in the literature in this connection.

40. Dr. T. H. Gronwall: *On Gibbs' phenomenon.* Second paper.

This paper contains certain theorems supplementary to those given in the paper read at the February, 1925, meeting of the Society, and also the numerical evaluation of the value of the Cesàro summation order above which Gibbs' phenomenon ceases to exist.

41. Mr. L. B. Robinson: *On equations in mixed differences.*

The author considers the equation $u'(x)=\sum_{j=1}^{n}P_j(x)u(a_jx+\delta_j)+Q(x)$, with $|a_j|<1$, $P_j(x)$ and $Q(x)$ polynomials, $u(x)=c_0+(c_1/\mu)x+(c_2/2\mu^2)x^2+\cdots$, $|\mu|<|\delta_j|(1+|a_j|+|a_j^2|+\cdots)$. The c's are given by solving the system $\mu_1=\lambda_{01}c_0+[1+\lambda_{11}]c^1+\lambda_{21}c_2+\cdots$, $\mu_2=\lambda_{02}c_0+\lambda_{12}c_1+[1+\lambda_{22}]c_2+\cdots$, $\mu_3=\lambda_{03}c_0+\lambda_{13}c_1+\lambda_{23}c_2+\cdots$, $\cdots$. The above system has a converging determinant, and the c's all satisfy the inequality $|c_i|<|M|$ $(i=1, 2, 3, \cdots)$, where M is some finite constant. The element c_0 is arbitrary. So the solution converges. He also considers the case $u'(x)=\sum_{j=1}^{n}(P_j(x)/x^k)u(a_jx+\delta_j)+Q(x)/x^k$, $\mu|a_j|+|\delta_j|<\mu$, $|a_j|\mu/\delta_j<1$. In this case, the solution has a logarithmic singularity at the origin, and a pole of order $k-1$.

42. Professor O. D. Kellogg: *Regular and exceptional boundary points for the general Dirichlet problem.*

Green's function for the general open continuum is defined, and some of its properties are developed. Regular and exceptional boundary points are defined as those at which Green's function does or does not approach zero, respectively. Simple derivations are given of important properties of Wiener's sequence solution of the Dirichlet problem, and some theorems on capacity are proved.

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