LOVITT ON INTEGRAL EQUATIONS

Linear Integral Equations. By William Vernon Lovitt. New York, McGraw-Hill Book Company, 1924. xiii+253 pages.

This book is designed as a textbook on the subject of integral equations for colleges and universities. It does not aim to be an exhaustive treatise. The book is very readable, is teachable in the main, and handles the subject in an interesting fashion. Of particular interest are to be found applications of integral equations to problems in heat, electricity, and sound.

The Volterra and Fredholm theories are developed in detail. The Hilbert-Schmidt theory is also treated at length, together with important applications. In addition, the early chapters terminate in sets of exercises, which while presenting little opportunity for mathematical talent, nevertheless furnish an opportunity for some drill in actually solving integral equations.

There are a number of misprints in the book, most of them easily recognizable and not very important. At the end of the first chapter are to be found some linear differential equations with constant coefficients. Initial values $y(0)$ and $y'(0)$ are prescribed not only for the function and its first derivative, but also the value of the second derivative of the unknown function is preassigned. It is easily seen, however, that $y''(0)$ cannot be arbitrarily preassigned, but is determined by the differential equation in terms of $y(0)$ and $y'(0)$. In the third example on page 8, the value of $y''(0)$ is not compatible with the value given for $y(0)$ and $y'(0)$. The answers to the first three problems of this exercise are incorrect. The answer to the fifth problem is correct, provided initial values $y(0)=1$, $y'(0)=1$ are assumed and the last coefficient of the differential equation is changed from 5 to 8. In problem 14, page 22, the given equation should have the second t under the integral sign changed to x, and the final equation given in the problem should have a minus sign on the right hand side. These corrections check with the values later given on page 68.

In Chapter III, the author makes use of the system of algebraic equations related to the given Fredholm equation. There is no attempt to pass to the limit, but the standard device is used of making the algebraic case merely serve as a guide. However, care must be taken to make the final guess at least plausible. For example, equation (9) at the bottom of page 26:

$$u(x) = f(x) + \frac{1}{D(\lambda)} \int_a^b f(t)\ D(x, t; \lambda)dt$$

is to be inferred as the limiting form of the solution from

$$u(t_k) = f(t_k) + \frac{1}{D(\lambda)} \int_a^b f(t)\ D(t_k, t; \lambda)dt,$$

which is in turn obtained from the limit of a sum. The discriminating reader will, however, regard this as legerdemain, since t_k is throughout the discussion the kth point of subdivision of the interval (a, b). Hence as n increases without limit, t_k approaches a and the limiting form which is obtained is

$$u(a)=f(a)+\frac{1}{D(\lambda)}\int_a^b f(t)D(a, t; \lambda)dt$$

instead of the desired equation (9). By the aid of the Duhamel-Moore theorem,* not only can such difficulties be obviated, but the actual passage to the limit can be encompassed. This the reviewer has carried through in using this book for class work.

An observation should be made concerning the discussion of Dirichlet's and Neumann's problems. The author obviously wishes to confine himself to the very simplest kind of plane region. He considers only regions with boundaries consisting of a curve with a continuously turning tangent at every point. It is to be noticed that without introducing any complexities, we may with the material at hand in the text, establish the same results for a region whose contour consists of a finite number of pieces of arcs with continuously turning tangents. The significance of this slight extension is apparent, when we consider that we have included the case of a region in the form of a square.

Finally, it would have been stimulating in a special text of this kind, to have included not only reference to treatises and other texts on this subject, but to have given references to original sources, especially those which have now become classical. In the preface, the author acknowledges his indebtedness to Professor Oscar Bolza (now of the University of Freiburg, formerly of the University of Chicago) for the free use of notes on a course delivered by the latter on *Integral equations* during the summer of 1913 at the University of Chicago.

The book as a whole is to be recommended as an excellent text for class use.

E. T. ETTLINGER

---

* Cf. R. L. Moore, *On Duhamel's theorem*, ANNALS OF MATHEMATICS, (2), vol. 13 (1912), pp. 161–166. Also H. J. Ettlinger, *A simple form of Duhamel's theorem and some new applications*, AMERICAN MATHEMATICAL MONTHLY, vol. 29 (1922), pp. 239–250.