THE CINCINNATI MEETING OF THE SOCIETY

The twentieth Western meeting of the Society was held at the University of Cincinnati on Friday and Saturday, December 28 and 29, in conjunction with the meeting of the Mathematical Association of America and the seventy-fifth meeting of the American Association for the Advancement of Science.

The meeting was attended by about one hundred persons, among whom were the following seventy-nine members of the Society:

Alexander, R. B. Allen, Archibald, G. N. Armstrong, Atchison, Babb, Bareis, Barnett, E. M. Berry, Borger, Boyd Bradshaw, Brand, Cairns, Cajori, A. D. Campbell, Carmichael, Coble, H. H. Conwell, H. T. Davis, Davisson, Denton, Dickson, Dostal, Dresden, Edington, L. C. Emmons, Eversull, Focke, Fort, Gerst, Hancock, E. R. Hedrick, Holl, Dunham Jackson, Kindle, Kuhn, Lefschetz, Le Stourgeon, MacMillan, March, Marshall, T. E. Mason, G. A. Miller, C. N. Moore, R. L. Moore, C. C. Morris, Muehlman, Nassau, Olson, Overman, Palmié, Plant, Rasor, C. N. Reynolds, H. L. Rietz, E. D. Roe, Roever, Rothrock, Safford, Schottenfels, W. G. Simon, Sinclair, Slaught, Edwin R. Smith, I. W. Smith, Spenceley, R. B. Stone, Swartzel, B. M. Turner, Vandiver, Van Vleck, Veblen, J. H. Weaver, Norbert Wiener, Wilczewski, Yanney, J. W. Young, Zehring.

The session of Friday afternoon was held under the joint auspices of the Society, the Mathematical Association, and Section A of the American Association for the Advancement of Science, and was presided over by Professor Hancock, Chairman of Section A. The program consisted of the following three papers:

I. Professor G. A. Miller (as retiring Chairman of Section A): *American mathematics during three quarters of a century.*

II. Professor A. B. Coble (as retiring Chairman of the Chicago Section of the Society): *On the equation of the eighth degree.*

III. Professor L. E. Dickson (by invitation of the Program Committee): *Algebras and their arithmetics.*

At the close of this session Professor Jackson moved and those present unanimously adopted a resolution expressing to the University of Cincinnati and in particular to its

department of mathematics, their appreciation of the cordial reception and the hospitality accorded the visitors.

On Friday evening a dinner was held at the Hotel Gibson for mathematicians and astronomers, at which 110 persons were present. Mr. J. A. Shohat, who has recently come from Russia, expressed the appreciation of Russian scientists for the aid received from America. Professor Slaught spoke on the incorporation of the Society, and on the changes made in the By-Laws. Professor Dresden reported on the progress of the campaign for an endowment of $100,000. President Veblen, after supplementing the remarks of the previous speakers, announced the establishment by the National Research Council of Fellowships in Mathematics.

Professor Jackson reported for the committee appointed to make arrangements for a symposium at the meeting of April 1924, that the symposium lecture would be given by Professor H. L. Rietz on *The mathematical theory of statistics.*

At the session of Friday forenoon President Veblen took the chair. On Saturday Vice-President R. L. Moore presided, relieved by Professors A. B. Coble and R. C. Archibald.

The following papers were presented at the sessions of Friday morning, and Saturday morning and afternoon. Dr. Kraupner was introduced to the Society by Professor Hancock, Mr. Merriman by Professor C. N. Moore. Dr. Wilder's paper was presented by Professor R. L. Moore. The papers of Professor Reynolds, Dr. Kraupner, Professors Chittenden, MacDuffee, Wahlin, Cajori, Emch, and Sisam, Mr. Vandiver, Professor Smith, and Miss Schottenfels were read by title.

1. Professor W. E. Edington: *Theory of construction of group generators as substitutions.*

In general, to prove the existence of an abstract group generated by two or more operators obeying certain conditions requires the exposition of substitutions which fulfil all the required conditions on the generators. In this paper a theory for constructing such substitutions is discussed. Several theorems are proved giving general rules for finding substitutions fulfilling in certain cases the conditions

$s_1^m = s_2^n = (s_1 s_2)^k = 1$, and in some cases additional conditions. The theory of transpositions is made the basis of this theory.

2. Miss Bess M. Eversull: *On the summability of the triple Fourier series at points of discontinuity of the function involved.*

In a paper previously presented to the Society (this BULLETIN, vol. 28 (1922), p. 289) it was shown that if a function of three variables is finite and integrable (Lebesgue) its Fourier development is summable ($C1$) to the value of the function at all points within the region of continuity of the function. It is the purpose of the present paper to show by similar methods that the Fourier development of a function satisfying the above conditions is in certain cases summable ($C1$) at points of discontinuity of the function and to determine the values to which the series is summable. Results have been obtained where the point of discontinuity is such that all other points of discontinuity in the neighborhood lie on a plane or curved surface through that point, on two planes or curved surfaces having certain relative positions whose intersection passes through the point, and on three planes parallel to the coordinate planes whose common point of intersection is the point of discontinuity.

3. Mr. G. M. Merriman: *On a general theorem regarding divergent series, and its application to the double Fourier series.*

In the present paper a general theorem is proved which gives sufficient conditions that a double series, known to be summable ($C1$), should also be convergent. By a combination of this theorem with that concerning the summability of the double Fourier series, certain sufficient conditions for the convergence of the double Fourier series are obtained; these conditions are analogous to Dini's conditions in the case of the ordinary Fourier series, and were first introduced by Miss Hilda Geiringer (MONATSHEFTE FÜR MATHEMATIK UND PHYSIK, vol. 29 (1918)). The methods employed in proving the above results are generalizations of those used by Mr. S. Pollard in carrying through a similar investigation in the case of the ordinary Fourier series (PROCEEDINGS OF THE LONDON SOCIETY, (2), vol. 15 (1916)).

4. Professor C. N. Moore: *On necessary and sufficient conditions for convergence factors in double series.*

Sufficient conditions that a set of factors introduced into the terms of a double series summable by Cesàro's method should cause the resulting double series to be convergent have been given by the author in a paper that appeared in the TRANSACTIONS OF THIS SOCIETY in 1913. In the present paper some of these conditions are modified in such a manner that the resulting set has the property of being both necessary and sufficient. Furthermore, it is shown that if certain additional restrictions are made, we obtain the necessary and sufficient conditions that the function defined by the series with the convergence factors should approach the value to which the original series is summable when the convergence factors approach unity.

5. Professor C. N. Reynolds, Jr.: *Note on the map coloring problem.*

In this note it is shown that in Dr. Franklin's recent solution of the map coloring problem for any map of n regions where $n < N$ and $26 \leqq N \leqq 42$ (AMERICAN JOURNAL OF MATHEMATICS, vol. 44 (1922)), the N is subject to the narrower restriction $26 \leqq N \leqq 32$.

6. Professor R. L. Moore: *Concerning the prime parts of a continuum which separates its plane.*

In this paper it is shown that if, in a plane S, M is a bounded continuum which has more than one prime part and no prime part of M separates S, then in order that $S-M$ should be the sum of two mutually exclusive connected domains having M as their common boundary it is necessary and sufficient that the set G whose elements are the prime parts of M should be a simple closed curve of prime parts in the sense that it is disconnected by the omission of any two of its elements which are not identical. For a definition of prime part ("Primteil") of a continuum, see Hans Hahn, WIENER SITZUNGSBERICHTE, vol. 130 (1921).

7. Dr. W. Kraupner: *Integral solutions of the diophantine equations* $\xi_1^2 + \xi_2^2 + \cdots + \xi_n^2 = \eta_1^2 + \eta_2^2 + \cdots + \eta_n^2$ *in the quadratic realm of rationality.*

This paper contains a generalization of the results ob-

tained by Hancock in his paper in Liouville's JOURNAL, (8), vol. 4 (1921), p. 327. It is shown that there exist an infinite number of integral solutions of the quadratic diophantine equations $\xi_1^2+\xi_2^2+\cdots+\xi_n^2=\eta_1^2+\eta_2^2+\cdots+\eta_n^2$. The investigation is based upon the irreducible quadratic equation $x^2-2a_1x-a_2=0$, a_1 and a_2 being rational integers, which defines an algebraic integer in the quadratic realm of rationality.

8. Professor Harris Hancock: *On algebraic equations whose roots are trigonometric functions.*

This paper considers algebraic equations whose roots are trigonometric functions, their discriminants, realms of rationality etc., and a method of derivation of certain reciprocal equations.

9. Professor E. W. Chittenden: *Properties of abstract sets implied by properties of the class of all continuous functions.*

Let P be an abstract class and $\delta(p,q)$ a non-negative function such that $\delta(p,p)=0$. The late Professor A. D. Pitcher studied the class C of all continuous real functions of a variable p with range P and obtained among other results the conditions implying that the space P be a compact, connected, metric space (A. D. Pitcher and E. W. Chittenden, *On the foundations of the calcul fonctionnel of Fréchet,* TRANSACTIONS OF THIS SOCIETY, vol. 19 (1918)).

The present paper proposes to extend and complete the results of Professor Pitcher. It considers the class C in the more general system (P, K) where K is a relation defining the points of accumulation of the subsets of P. The paper contains a set of independent properties of C, a characterization of those spaces P which admit a non-constant continuous function, and certain generalizations and modifications of previous results. It is found that with respect to the theory of continuous functions the notions point of accumulation and neighborhood (voisinage) are equivalent.

10. Professor A. D. Campbell: *The classification of linear families of conics in various domains.*

The domains considered in this paper are the ordinary complex domain, the real domain, and the Galois fields of order p^n $(p>2)$. The corresponding planes considered are

the complex projective plane, the real projective plane, and the Galois field projective plane. The classes of conics and of pencils of conics in these planes are well known; as also the nets and three- and four-parameter linear families of conics in the complex domain, and nets in the above Galois fields.

This paper deals with the nets and three- and four-parameter families in the real domain, also the three- and four-parameter families in the Galois fields. A geometrical description of all these classes is given by means of apolarity. The classes of families in the other domains are obtained as sub-classes of those in the complex domain or by interpretation therefrom. The classes of three- and four-parameter families are obtained by apolarity from the classes of pencils and the types of conics respectively. The classes of nets of conics are obtained by use of the associated cubic curves in the plane of the parameters of the nets.

11. Professor Mary E. Sinclair: *The isoperimetric problem with variable end points.*

Using the derivatives of the extremal-integral, necessary conditions are found for a minimum in the usual isoperimetric problem with variable end points. These conditions, strengthened in the usual manner, are found to be sufficient for a weak minimum. Extension to the isoperimetric problem is made of the Bliss geometric interpretation of critical value of radius of curvature. If r_1, r_2, C_1 and C_2 are radii and centers of curvature of the fixed curves at the two end points, P_1 and P_2, there exists for every value of r_1 a critical value $\bar{r}_2$ of r_2, increasing with r_1, projective to r_1, and such that, according as P_1P_2 and $P_2\bar{C}_2$ have the same or opposite direction, C_2 must or must not lie on $P_2\bar{C}_2$.

Our conditions may be expressed in a form very close to those of Merrill for his special type of problem. Again, as with Dresden's results in the non-isoperimetric case, they can be expressed in terms of particular solutions of the differential equation, $\Psi(w) + \mu V = 0$, and they reduce to Dresden's if the isoperimetric condition is dropped.

12. Professor Mary E. Sinclair: *The hanging chain with end points variable on curves in a plane.*

The conditions for a minimum in the isoperimetric problem with variable end points are applied to the problem of the

hanging chain of given length and uniform density. In the cases of one and of two variable end points a geometrical construction is given for the critical value, $\bar{r}_2$, of the radius of curvature.

13. Professor C. C. MacDuffee: *Covariants of differential forms of arbitrary order and degree.*

Covariants of differential forms of arbitrary order and degree have been studied by E. Pascal, ATTI DEI LINCEI, MEMORIE FISICHE, vol. 8 (1910). The definition of differential form used in this paper is essentially equivalent to that given by Pascal, but the subsequent treatment is quite different. A differential form of order h and degree r is defined as a polynomial of the type

$$\sum a_{i_1 i_2 \cdots i_m j_1 j_2 \cdots j_m} d^{i_1} x_{j_1} d^{i_2} x_{j_2} \cdots d^{i_m} x_{j_m},$$

the sum extending over all distinct partitions of r into positive summands $i_1, i_2, \ldots, i_m$, each $\leq h$, and $j_1, j_2, \ldots, j_m$ being chosen with repetitions allowed from among the integers $1, 2, \ldots, n$, so as to yield all possible distinct terms. The coefficients a are functions of $x_1, x_2, \ldots, x_n$. The principal theorem obtained, an analogue of Hilbert's theorem, is that every relative covariant of weight ε in at most n distinct differential operators can be expressed as a function with constant coefficients of certain elementary covariants and their differentials multiplied by the εth power of the identical covariant $| dx_i |$. These elementary covariants are roughly analogous to the polars of the ground forms in the algebraic theory.

14. Professor G. A. Miller: *Number of cycles of the same order in any substitution group.*

This paper appears in the present number of this BULLETIN.

15. Professor I. A. Barnett: *Note on linear differential equations with constant coefficients.*

This note considers linear differential equations in general analysis whose coefficients are constant with respect to the independent variable. It is shown that in case the coefficient system is either symmetric or skew-symmetric the constituents of the fundamental set of solutions are all of the form of exponentials and not products of exponentials by powers of the independent variable. The result for the

symmetric case is given without proof in a course of lectures of Professor Bôcher, and the statement is made that Weierstrass completely solved this case in 1858, ten years before he introduced elementary divisors.

16. Professor H. W. March: *Deflection of a rectangular plate clamped at its edges.*

The solution of the problem of the deflection of a rectangular plate clamped at its edges and under a given load has not previously been expressed in terms of functions which satisfy either the prescribed non-homogeneous differential equation or the corresponding homogeneous equation. W. Ritz (Crelle's JOURNAL, 1909) gave a solution in terms of a series of functions which satisfy the boundary conditions but which do not individually satisfy either differential equation.

The classical method of treating such problems has failed in this case because it has not been found possible to combine known solutions of the homogeneous equation with a solution of the non-homogeneous equation in such a way as to satisfy all of the boundary conditions. In this paper an apparently new type of solution of the homogeneous equation is given and the problem in question is solved for the case of a uniform load. It appears that the method can be readily extended to more general cases. The present method gives a result that is much better adapted than that of Ritz to numerical computation and to the discussion of the elastic surface.

17. Dr. Norbert Wiener: *A generalization of the Dirichlet problem.*

It is shown that the Dirichlet problem is a particular case of a problem solvable for any bounded open region and any continuous boundary conditions.

18. Professor Arnold Dresden: *On Brouwer's contributions to the foundations of mathematics.*

This paper has appeared in the January-February number of this BULLETIN.

19. Professor G. E. Wahlin: *On the application of the theory of ideals to diophantine equations.*

This paper has appeared in the March-April number of this BULLETIN.

20. Professor Florian Cajori: *J. H. Rahn's mathematical symbols.*

The symbols in Rahn's Algebra written in German (1659) are compared with those found in the English translation (1668). There are set forth certain incorrect statements due to John Collins and John Wallis regarding John Pell's rôle in the preparation of the original edition as well as of the translation.

21. Professor Dunham Jackson: *A generalized problem in weighted approximation.*

In a recent note in this BULLETIN (June, 1923), the author has discussed the convergence of the trigonometric sums $T_n(x)$ determined as approximations to a given continuous function $f(x)$ by the condition that the integral of the square of the error, multiplied by a weight-function $\varrho(x)$, shall be a minimum. The function $\varrho(x)$ was assumed to be bounded, and to have a positive lower bound. The purpose of the present paper is to deal with the somewhat less simple problem of convergence that arises if the greatest lower bound of $\varrho(x)$ is zero. There is occasion incidentally to develop the theory of the existence and uniqueness of discontinuous approximating functions of a high degree of generality. The discussion is put in such form as to be applicable when the square of the error is replaced by the mth power of its absolute value.

22. Professor E. B. Van Vleck: *On the zeros of polynomials.*

In a very interesting memoir in the ANNALES DE L'ÉCOLE NORMALE SUPÉRIEURE, 1923, Montel shows, among other results, that, when the first $r+1$ coefficients $1, a_1, \ldots, a_r$ of the equation

$$1 + a_1x + a_2x^2 + \cdots + a_rx_r + a_{r+1}x^{r+r_1} + \cdots + a_{r+n}x^{r+r_n} = 0 \quad (r_i \geqq i)$$

are given, the r roots of smallest absolute value lie within a circle whose radius depends only upon r and the number of terms $n+r+1$ in the equation, and he determines this radius for $r=1$ and $a_1=1$, and for $r=2$, $a_1=0$. In this paper these results are supplemented by the determination of such a radius for any value of r when each $r_i=i$. It is also ascertained under what conditions an arbitrarily selected set of r coefficients a_i will limit in absolute value the r smallest roots and their upper limit is then determined when the degree of the equation is given.

23. Professor Arnold Emch: *On the Weddle surface and analogous loci.*

This paper starts from the geometric definition of the Weddle surface as the locus of the vertices of all quadric cones through six points in space. The locus of the vertices of quadric cones through seven points is a sextic S of genus 3. All quadrics through the seven points pass through an eighth fixed point; hence the sextic S is also the locus of vertices of quadric cones through the 8 base points of the net of quadrics. S cuts each of the 28 lines, connecting the base points, in two points. There are 28 Weddle surfaces through S, each having 6 of the 8 base points as nodes. The problem is also investigated for the cubic and Geiser transformations connected with the 8 base points.

The locus of the vertices of cubic cones through 10 points B is found to be a surface of order 15 with the B's as cubic nodes, passing through each of the 45 lines joining the B's. The locus of the vertices of cubic cones through 11 points is a curve of order 189 cutting each of the 45 lines in 12 points; its genus is determined. Finally the paper gives the loci of the vertices of $n$ic cones through $(n(n+3)/2)+1$ and through $(n(n+3)/2)+2$ points in space.

24. Professor Tomlinson Fort: *Note on Dirichlet and factorial series.*

In a note published in the TRANSACTIONS OF THIS SOCIETY, January, 1922, the author studied a class of series which included ordinary Dirichlet series and factorial series as special cases. The present paper extends the results of that note.

25. Dr. R. L. Wilder: *On the dispersion sets of a connected point set.*

A proper subset H of a point set M is called a dispersion set of M if $M-H$ is totally disconnected. H is called a primitive dispersion set of M if no proper subset L of H exists such that $M-L$ is totally disconnected. A special study is made of the properties of connected sets that contain primitive dispersion sets. If H is a primitive dispersion set of a connected set M consisting of a finite number of points (the existence of connected sets with primitive dispersion sets consisting of a finite number of points

has recently been established by Knaster and Kuratowski, (FUNDAMENTA MATHEMATICAE, vol. 2 (1921), pp. 206–255), it is shown that M is the sum of a finite number of mutually exclusive connected sets, each of which consists of more than one point and contains one and only one point of H. It is also shown that if a connected set M contains a primitive dispersion set H which consists of a finite number of points, then H is the only primitive dispersion set of M.

26. Mr. J. A. Shohat: *The theory of closure of Tchebychef's polynomials for an infinite interval.*

If $\varphi_n(x)$ $(n = 0, 1, 2, \ldots)$ denote a system of Tchebychef's orthogonal and normal polynomials corresponding to the interval (a, b) with the characteristic function $p(x)$ not negative in (a, b), then $\int_a^b p(x)\Phi_m(x)\Phi_n(x)dx$ is equal to unity if m and n are equal, and to zero if m and n are unequal. In the case of a finite interval (a, b), every function $f(x)$ which with its square is integrable in (a, b) satisfies an important *closure equation.*

In this paper, the author investigates the closure equation for an infinite interval (a, ∞), and shows that the closure equation holds if, for x sufficiently large, $p(x) < e^{-|x|^\lambda}$, with $\lambda > 1/2$ for $a = 0$, and $\lambda > 1$ for $a = -\infty$. Particular cases are the polynomials of Hermite-Tchebychef and those of Laguerre-Tchebychef. The closure equation is not true in general. A general example is given of a system of Tchebychef's polynomials in $(0, \infty)$ for which the closure equation is not true.

27. Professor C. H. Sisam: *On curves whose first polars contain a pencil of lines.*

The algebraic plane curves for which every right line through a fixed point is a component of a first polar with respect to the curve are completely determined.

28. Professor H. T. Davis: *Integral equations as differential equations of infinite order.*

In this paper an equivalence is pointed out between linear differential equations of infinite order and integral equations of either the Volterra or Fredholm type which have solutions analytic within the interval of integration. An existence theorem for the differential equation of infinite order with constant coefficients in both the homogeneous and non-

homogeneous cases is given and the solutions are shown to satisfy a singular integral equation of Volterra type.

29. Professor L. E. Dickson: *On the theory of numbers and generalized quaternions.*

This paper opens with a new simple proof by the method of descent of Bachet's theorem that every positive integer is a sum of four integral squares, and the new generalization that the roots of the four squares may be chosen to satisfy four assigned linear congruences.

Next there is given a simpler proof of the author's formulas for all integral solutions of $x^2+y^2+z^2+w^2=uv$. These formulas and generalized quaternions occur in the author's *Algebras and Their Arithmetics*, University of Chicago Press, 1923, pp. 187–198. The present paper discusses an arithmetic of generalized quaternions for which there exists a greatest common left divisor, although it cannot be found by a process of division with remainders of decreasing norms. The new method is first applied to the arithmetic of quaternions and shows the difference between the integral quaternions of Lipschitz and those of A. Hurwitz. Application is made to diophantine equations. The memoir will appear in the AMERICAN JOURNAL.

30. Professor L. E. Dickson: *Quadratic fields in which factorization is always unique.*

This paper appears in this number of this BULLETIN.

31. Professor A. B. Coble: *Geometric interpretation of the expression of an algebraic form as a determinant.*

Dickson and Everett have discussed the possibility of expressing a given algebraic form in k variables as a determinant whose elements are forms of the same order. It is the purpose of this paper to show that this possibility rests when $k=3$ upon the existence of coresidual point groups on the given algebraic curve; when $k=4$ upon the existence of coresidual linear systems of curves on the given algebraic surface; etc. If the determinant is to be symmetric, contact systems come into play.

32. Mr. H. S. Vandiver: *Sets of three consecutive integers which are quadratic or cubic residues of primes.*

In this paper the author proves several theorems such as the following: If p is of one of the forms $40k+1$,

$40k+11$, $40k+29$, $40k+39$, then $a+1$, a, $a-1$ are all quadratic residues of p, where $a^2 \equiv 5 \pmod{p}$ and 5 is a biquadratic residue of p.

33. Mr. H. S. Vandiver: *On Kummer's memoir of* 1857 *concerning Fermat's last theorem.*

In two previous papers under this title the author pointed out that Kummer made some errors in the proofs of certain theorems. In the first paper (PROCEEDINGS OF THE NATIONAL ACADEMY, vol. 6 (1920), pp. 266–269) these theorems were numbered I to IV. The second paper (this BULLETIN, vol. 28 (1922), pp. 400–407) contained proofs of Theorems I and IV. The present paper is devoted to the proof of Theorem III and an extension of Theorem II is also proved which enables the author to establish the result: If the class numbers of the field $\Omega(\alpha)$, $\alpha = e^{2i\pi/p}$, is divisible by p but not by p^2, then $x^p+y^p+z^p = 0$ is not satisfied in integers in the field $\Omega(\alpha+\alpha^{-1})$, not zero, p being an odd prime. This includes Kummer's results as special cases.

34. Professor H. L. Smith: *Necessary and sufficient conditions for the existence of a class of Stieltjes integrals.*

In this paper necessary conditions are obtained for the existence of the Stieltjes integral $\int_0^1 \psi(t)d\varphi(t)$ which are also sufficient if the associated curve $x=\varphi(t)$, $y=\psi(t)$ $(0\leqq t\leqq 1)$ is continuous and such that its multiple points are a reducible set. The conditions remain sufficient for discontinuous curves which can be embedded in continuous curves of a certain character. A corollary is that every simple closed (continuous) curve is squarable if the line integral $\int ydx$ exists over it. An example is given of a simple closed squarable curve over which the line integral $\int ydx$ does not exist.

35. Professor S. Lefschetz: *On polyhedra in euclidean n-space.*

In this paper are first developed certain properties of covering manifolds, which are then applied to the proof of this theorem: A bounded, non-singular, $(n-1)$-dimensional polyhedron in euclidean n-space bounds a finite region which together with the polyhedron is homeomorphic to an n-cell plus its boundary.

36. Professor R. L. Moore: *An extension of the theorem that no perfect set is countable.*

Sierpinski has shown that if the closed and bounded point set M is the sum of a countable number of mutually exclusive closed point sets $M_1, M_2, M_3, \ldots$ then M is not connected. In the present paper it is shown that, if the additional requirement is imposed that the point sets $M_1, M_2, M_3, \ldots$ should be connected, then not every one of them contains a limit point of the sum of all the others. It is shown that this latter conclusion does not hold if either this additional requirement or the requirement that M be bounded be omitted.

37. Professor Louis Brand: *Note on the integral theorems of vector analysis.*

The integral theorems of vector analysis fall into two principal groups. In the first group the volume integral of $\nabla\omega$ where ω denotes a scalar, vector, or dyadic point function is transformed into an integral of $\boldsymbol{u}\omega$ over the bounding surface, $\boldsymbol{u}$ being the unit surface normal. In the present note it is shown that the second class of theorems, which transform surface to circuit integrals, may be deduced from the first class. The proof hinges on the fact that $\boldsymbol{u}$ may be regarded as the gradient of a scalar point function whose level surfaces are parallel to the original surface.

38. Miss I. M. Schottenfels: *The Kürschak field of complex numbers.*

The author extends to the complex number field the results obtained by J. Kürschak in his paper in Crelle's JOURNAL, vol. 142 (1913). The work of Ostrowski (ACTA MATHEMATICA, vol. 41 (1918)) is also reorganized and extended.

39. Miss I. M. Schottenfels: *The error in Hartog's proof of the Zermelo theorem.*

The author points out an incompleteness in Hartog's proof of Zermelo's theorem which appeared in MATHEMATISCHE ANNALEN, vol. 76 (1915) and furnishes a complete proof, far simpler and more concise than those published by Kuratowski in volume 2 of the FUNDAMENTA MATHEMATICAE.

ARNOLD DRESDEN,
Assistant Secretary.