introduced by Darboux, and who has appreciated its power in dealing with geometrical problems, must wonder why Bianchi has never made much use of it. He introduced the elements of it in his treatment of Weingarten's method for the study of applicable surfaces (vol. II, p. 176), but seems not to have used it elsewhere. It appears as a note at the close of this volume.

Bianchi is a past-master in the art of writing treatises, not only in the field most closely associated with his name, but in many other fields. And this book reveals him at his best. The reader has no grounds for criticizing the book as to clarity of statement, but the student who wishes to follow up a particular subject may regret the lack of references to many of the sources from which the cream has been taken.

L. P. EISENHART

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# THE IMUK REPORTS AND LOREY ON INSTRUCTION IN GERMAN UNIVERSITIES

*Das Studium der Mathematik an den deutschen Universitäten seit Anfang des 19. Jahrhunderts.* Von W. Lorey. (Abhandlungen über den Mathematischen Unterricht in Deutschland veranlaßt durch die Internationale Mathematische Unterrichtskommission.* Band III, Heft 9). Leipzig, Teubner, 1916. 12 + 431 pages and 4 plates.

During the deliberations of the fourth International Congress of Mathematicians at Rome in 1908, steps were taken to organize an International Commission on the Teaching of Mathematics, the members of which were to prepare or procure reports on the methods and materials of mathematical instruction in different countries. Most of these reports were ready when the fifth International Mathematical Congress convened at the University of Cambridge in 1912, but several more have appeared since then. At this writing, 18 countries have published 294 reports containing over 13,500 pages. Germany has issued 53 reports with a total of 5571 pages; about one-fourth of this space is required by the United States for its 18 reports, and about one-sixth of the same space by each of the following countries: Austria for 13 reports; Great Britain for 39; Switzerland for 13; and Japan for 16 reports in two volumes. The reports of France cover nearly 700 pages. Of more modest dimensions are, in order of size, the reports from Belgium, Russia (including Finland),

* A new word *IMUK* has been coined and is used in German writings as an abbreviation for these last three words.

Hungary, Italy, Sweden, Spain, Holland, Denmark, Australia, Argentine, and Roumania (one report of 16 pages).

The German reports have been published under the general heading: Schriften des deutschen Unterausschusses der IMUK. They include Abhandlungen, edited by F. Klein, and Berichte und Mitteilungen, edited by W. Lietzmann, of which the first series appeared in 12 numbers during the years 1909–1916 and extends to 356 pages. Two numbers of the second series are the reports on mathematical instruction in Denmark (1915; 60 pp.) and England (1915, 211 pp.).* The third and last volume (1917, 115 pp.) of this second series contains a complete index to both the Berichte und Mitteilungen and the Abhandlungen.

The Abhandlungen (38) by 33 different authors (1909–1916), and containing over 4800 pages, are now complete and occupy five (in reality nine)† volumes. The general titles of these volumes are as follows: 1. Die höheren Schulen in Norddeutschland; 2. Die höheren Schulen in Süd- und Mitteldeutschland; 3. Einzelfragen des höheren mathematischen Unterrichts; 4. Die Mathematik an den technischen Schulen; 5. Der mathematische Elementarunterricht und die Mathematik an den Lehrerbildungsanstalten. The imposing Abhandlung under review is the last number of the third volume and it was the last one of all the Abhandlungen to be published‡ (preface dated "6. Juli 1916"). It is more than twice as large as any of the others.

For the American mathematician this volume will probably be by far the most interesting of the Abhandlungen. As in nearly all the other reports, the historical side is heavily emphasized; but here movements and developments are associated with institutions and men whose names are more or less familiar. Moreover the biographical outlines (some quite complete) provided in the case of every mathematician mentioned, are occasionally entertainingly supplemented by extracts from published sketches,§ and reminiscences especially prepared for this volume. Only two days before his death Heinrich Weber contributed an account of his student days.

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* This volume is decidedly useful for filling up many lacunae of the reports from Great Britain which are confined almost wholly to an account of conditions in England.

† Each of three Bände is divided into two Teile, and one of the Teile has two Abteilungen.

‡ A number of the countries have made no report on mathematical instruction in their universities, except in connection with account of the training of secondary school teachers.

§ The bibliographical references are numerous and valuable; much material rarely met with has been brought to light and placed conveniently for reference.

Interesting facts are given concerning eminent mathematicians, other than German, who influenced German mathematics; for example, Abel, Euler, Lie, Lagrange, and Steiner. Regarding such a man as Gauss, the formost German mathematician of the nineteenth century, there is very little, since the volume aims to give an account of mathematical instruction in the universities, and not a history of the development of mathematics as a science. Gauss was director of the observatory and professor of astronomy, not mathematics, at the University of Göttingen, from 1807 to the time of his death in 1855, and he "hat immer nur vor einem kleineren Kreise von Zuhörern und nie mit großer Freude gelesen."

The first chapter of 22 pages contains general information about the 22 German universities. Six of these were founded in the fifteenth century, the one at Leipzig, established in 1409, being the oldest. Four universities were founded in the nineteenth century, while that at Frankfort-on-Main was opened in the autumn of 1914. Just half of the universities are in Prussia.

There are sections on the faculties, the teaching force, and the students. Of the seven classes of teachers comprised in the 3724 professors and instructors for the wintersemester of 1914–15, about 3400 were ordinary and extraordinary professors and privatdozenten. The first and third of these groups were almost numerically equal and constituted about four-fifths of the whole. In mathematics there were 49 ordinary and 19 extraordinary professors and 22 privatdozenten. Of these 90 teachers Göttingen had 9, Munich 7, Strasbourg 6, and so on down to Rostock with 2.

The section on students contains two charts; one indicates the number of candidates for the Prussian Staatsexamen, with mathematics and natural science as major, between 1839 and 1913. From about 20 in 1839 there was a fairly constant increase to nearly 160 in 1882; this number dropped to less than 25 by 1893, but increased almost continuously to over 325 by 1913. The second chart shows the number of students of mathematics and natural science at the University of Göttingen from 1868 to the summer of 1913. The numbers increased from about 60 in 1870 to almost 240 in 1881, dropping to less than 80 in 1892 but increasing to over 750 by 1913. The intimate relation between the number of university students and the number of those taking the Staatsexamen is noticeable.

The rest of Lorey's work is divided into five chapters, 2–6. The second (26 pages) deals with the state of mathematical instruction in the universities at the beginning of the nineteenth century, and with vain trials of methodically arranged new organization in the years 1818–1848. The third chapter (pages 59–111) is entitled: "The development of mathematical instruction in the universities up to the

founding of the German Empire." The fourth chapter (pages 111-141) discusses "Special achievements in the period before 1870 with glances at the present time." The fifth chapter (pages 142–211) deals with "The period from 1870 to 1890. From the founding of the Empire to the beginning of the school reform movement." The sixth chapter (pages 211–402) is entitled: "Recent times (from 1890–1914)." Pages 403-8 are filled with "Corrections and additions" and pages 409–28 with a "Personen-Verzeichnis".

Although lectures in mathematics were given in German universities in the latter part of the eighteenth century, they were not universally regarded as feasible even by those who flourished in the early nineteenth century. A notable case is that of Mollweide (a friend of Gauss who characterized him as "ein gründlicher Mathematiker und trefflicher Mensch"), who did not think that higher mathematics could be properly dealt with in lectures "schon weil es dabei zuviel Schreibens an der Tafel gäbe!"

In the early part of the nineteenth century the universities at Königsberg (founded in 1544), Berlin (1810), and Göttingen (1737) were the three chief centers of mathematical inspiration. The first named university, especially, under the guidance of such notables as K. G. J. Jacobi, Bessel, and Franz Neumann, led to a well defined Königsberg school whose influence throughout Germany was great. For the 17 years 1826–42 Jacobi was constantly giving in his lectures results developed by prolific research. With Neumann he founded in 1834 a mathemathics-physics seminar, concerning which full and interesting details are given in the section on seminars in Chapter 4. About 1843 Jacobi moved to Berlin where he lived as a royal pensioner until his death in 1851. Jacobi's first pupil Richelot, a native of Königsberg, became a professor in the university and expounded the researches of his teacher and of Steiner. L. A. Sohncke and Otto Hesse were also natives of Königsberg. While the former served only two years as dozent in the university, the latter taught there as dozent and professor for 15 years. The influence of the Königsberg school was later extended when Hesse, Gustav Kirchhoff, H. von Helmholtz, and Bunsen were teaching together at Heidelberg University which "in ihrer höchsten Blüte stand". It was at this time that Heinrich Weber was a student, and his interesting reminiscences of the period have been preserved by Lorey.

Weber's early training doubtless influenced him in accepting the call to the University of Königsberg where he was professor for the ten years 1873–83.

In Berlin by 1835, Crelle (1780–1855), Lejeune Dirichlet (1805–1859), Dirksen (1792–1850), a pupil of Gauss, and Steiner (1796–1863) were prominent mathematical members of the Academy of Sciences. A little later Jacobi was also there. It was under Dirksen at the University

in 1825 that Jacobi, when only twenty years of age, passed a brilliant doctor's examination. Although Steiner had studied no mathematics and could scarcely even write at the age of 16 years, he rose rapidly into prominence by his publications during the years 1825-34 that he was teacher in the Gewerbeschule at Berlin. In 1832 on the recommendation of Bessel and Jacobi the philosophical faculty of Königsberg university conferred the honorary degree of doctor on Steiner. He was elected a member of the Berlin Academy in 1833, and became professor at the University of Berlin in 1834. He held this position until his death.

Of the three leaders (Dirichlet, Steiner, and Jacobi) of the first mathematical school at Berlin, Steiner undoubtedly exercised the greatest influence on his students and on geometrical instruction in schools. A great mass of ideas and propositions discovered or employed by Steiner penetrated into the teaching quite quickly, such as the theory of harmonic points and pencils, of the potency of a point, of pole and polar, the properties of the complete quadrilateral, and Pascal's Theorem.

Dirichlet was born in Aix-la-Chapelle. When 16 years of age, after finishing his secondary school education at Cologne, he went to Paris, where he studied mathematics and taught for six years. Through Fourier he met Alexander von Humboldt, whose influence with the Ministry* resulted in his appointment as dozent in the University of Breslau in 1827. In 1829 he was called to the University of Berlin where he finally became ordinary professor in 1839. He remained there until about four years before his death, when he became Gauss's successor at Göttingen.

Crelle was not a professor in the university but a "Geheimer Baurat" who was, for example, responsible for the plans of the railway between Berlin and Potsdam. His numerous mathematical publications were not of the first rank, and his significance for mathematics lay in his energetic organizing activity. With Gergonne's ANNALES DE MATHÉMATIQUES PURES ET APPLIQUÉES as a model, he founded the JOURNAL FÜR DIE REINE UND ANGEWANDTE MATHEMATIK in 1826. Success in the undertaking was partly assured through an arrangement which he made with the Kultusministerium (ministry of ecclesiastical affairs and public instruction) whereby he was allowed to add to the title of the JOURNAL the words to be found even in recent volumes: "Mit thätiger Beförderung† hoher Königlich Preussischer Behörden." This Beförderung consisted on the one hand in issuing strong official recom-

* Although Alexander von Humboldt (1709-1859) was not a mathematician his name comes up several times in the Abhandlung as that of one very especially interested in the subject and wielding his powerful influence for the promotion of its study.

† For "thätiger Beförderung" Lorey has "Unterstützung".

mendation of the JOURNAL, not only to universities and higher schools, but also, for example, to government boards and, through Prussian ambassadors, in foreign countries. On the other hand the Beförderung involved the purchase of a number of copies of the JOURNAL which were distributed to various schools—a custom prevailing until the great war. In this way, Weierstrass, for example, while a gymnasium pupil, received inspiration by discovering an uncut copy of the JOURNAL "mit den schönen Abhandlungen von Steiner, von denen auch ein Primaner etwas verstehen konnte."

But in connection with the JOURNAL, the important thing was that Crelle's faith in young collaborators* was not misplaced. "Darin liegt gerade das unvergängliche Verdienst des damals schon bald fünfzigjährigen Crelle, daß er die jungen mathematischen Forscher, die in den zwanziger Jahren des vorigen Jahrhunderts in Deutschland auf einmal hervortraten, an sich heranzog und ihnen in seinem JOURNAL eine leichte Publikationsmöglichkeit verschaffte."

These indications as to the contents of the volume will suffice for suggesting its general character. It is an invaluable work of reference on the subject of which it treats.

R. C. ARCHIBALD

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* The first volume contained seven memoirs by Abel, five by Steiner, and one by Jacobi.

In spite of the title of the JOURNAL very few articles on applied mathematics were ever published in it. In 1829 Crelle founded another periodical JOURNAL FÜR BAUKUNST, which existed until 1851.