BIANCHI'S DIFFERENTIAL GEOMETRY

Lezioni di Geometria Differenziale. Terza edizione interamente refatta
in due volumi. Vol. I. By L. Bianchi. Pisa, Spoerri, 1921. IV + 806 pp.*

The first and second editions of this classic, whether in Italian or
German, are so well known to all students of differential geometry,
or should be, that an extended review of this edition would be an
anachronism. When one turns over the pages he is reminded of the
earlier editions, but when he compares it with the latter he observes
much that is new in the development of the topics formerly treated.
And so he decides that hereafter, when he wants to see what Bianchi
says about a certain subject (and a student of differential geometry
cannot afford not to do so), he will use the new edition.

The first ten chapters of the second editions, both Italian and
German, are retained in this volume, but they have been developed to
such an extent that they occupy half again as many pages.

The problem of reducing the linear element of a surface to the form
$ds^2 = du^2 + 2 \cos \omega \, du \, dv + dv^2$ is called the *problem of Tschebychef*
by Bianchi, and the parametric curves the *lines of Tschebychef*. This
problem was discussed briefly on page 401 of Volume 2 of the old
edition, but the new treatment is more extensive. In fact, it is shown
that such lines exist on any surface, the degree of arbitrariness being
that of two arbitrary functions of one variable each (p. 159). More
recently Bianchi† has proved that the tangents to either family of
these lines, say $v = $ const., at points of any curve of the other family are
parallel in the sense of Levi-Civita with respect to the latter curve;
moreover, this is a characteristic property of the lines of Tschebychef.

In Chapter V it is shown that the Christoffel symbol of the second
kind formed with respect to the second fundamental form is the arith-
metic mean of the corresponding symbols formed with respect to the
linear element of the surface and of its spherical representation. Also
there are certain results concerning spaces of constant Riemannian
curvature of n dimensions (p. 259).

In 1869 Dini solved the problem of geodesic representation of two
surfaces upon one another. In 1896 Levi-Civita extended the problem
to spaces of any order. The results of Dini and their application to

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the determination of pairs of quadrics in this relation have been added
to Chapter VI.

When two surfaces are applicable, the curves on one corresponding
to asymptotic lines on the other have been called *virtual asymptotic*
by Bianchi. The differential equations determining such lines on any
surface were derived by Darboux, who showed that the problems of
finding these lines on a surface S and the surfaces applicable to S
are equivalent. This theory is set forth at the close of Chapter VII;
it appears at the same place in the second German edition but not in
the Italian one.

In his beautiful theory of surfaces applicable to a quadric Bianchi
made use of the theorem of Chieffi: If at the point of any asymptotic
curve C upon a surface S applicable to a ruled surface R one draws
tangents to the geodesics g of S which are deforms of the generators
of R, the ruled surface R_1, formed by these tangents is applicable to
S with g remaining rigid. Bianchi adds in § 153 an analytic proof of
this theorem to the geometric proof given on page 4 of volume 3 and
in § 126 of the second German edition.

Twelve years ago Sannia published several papers in the RENDICONTI
DI PALERMO dealing with the intrinsic definition of rectilinear con-
gruences by means of two quadratic forms. Bianchi devotes §§ 187
and 188 to this theory and to an application to congruences whose
mean evolute is a point.

The general theory of relativity has aroused a general interest in
the differential geometry of Riemann spaces of any order. The last
half of the first volume of the second Italian edition contains an ex-
cellent exposition of this theory. This is omitted from the volume
under discussion with the exception of the chapter dealing with "pseudo-
spherical geometry and its non-euclidean interpretation", which appears
as Chapter XIV. However, the last half of the second volume will
be devoted to general Riemann geometry, and we can expect that it
will contain some of the recent additions to this live subject.

Chapter XI is devoted to surfaces with plane or spherical lines of
curvature, which appeared formerly as Chapter XXI with some omissions
and additions. Chapters XXII and XXIII of the second Italian edition,
or XIV and XV of the German edition, dealing with minimal sur-
faces, appear with little change as Chapters XII and XIII.

Bianchi began his career as a student of pseudospherical surfaces, and
the theory of these surfaces and their transformations owes to him its
present state of perfection. This subject has always seemed particularly
fascinating to him. And now here in the last two chapters of this volume
we have his exposition of surfaces of constant curvature based upon the
knowledge and experience of a lifetime. They are extremely well done.

Anyone who is familiar with the method of the moving trihedral,

introduced by Darboux, and who has appreciated its power in dealing
with geometrical problems, must wonder why Bianchi has never made
much use of it. He introduced the elements of it in his treatment of
Weingarten's method for the study of applicable surfaces (vol. II,
p. 176), but seems not to have used it elsewhere. It appears as a
note at the close of this volume.

   Bianchi is a past-master in the art of writing treatises, not only
in the field most closely associated with his name, but in many other
fields. And this book reveals him at his best. The reader has no
grounds for criticizing the book as to clarity of statement, but the
student who wishes to follow up a particular subject may regret the
lack of references to many of the sources from which the cream has
been taken.

<div align="right">L. P. Eisenhart</div>

THE IMUK REPORTS AND LOREY
ON INSTRUCTION IN GERMAN UNIVERSITIES

Das Studium der Mathematik an den deutschen Universitäten seit An-
   fang des 19. Jahrhunderts. Von W. Lorey. (Abhandlungen über den
   Mathematischen Unterricht in Deutschland veranlaßt durch die Inter-
   nationale Mathematische Unterrichtskommission.* Band III, Heft 9).
   Leipzig, Teubner, 1916. 12 + 431 pages and 4 plates.

   During the deliberations of the fourth International Congress of
Mathematicians at Rome in 1908, steps were taken to organize an
International Commission on the Teaching of Mathematics, the members
of which were to prepare or procure reports on the methods and materials
of mathematical instruction in different countries. Most of these reports
were ready when the fifth International Mathematical Congress convened
at the University of Cambridge in 1912, but several more have appeared
since then. At this writing, 18 countries have published 294 reports
containing over 13,500 pages. Germany has issued 53 reports with a
total of 5571 pages; about one-fourth of this space is required by the
United States for its 18 reports, and about one-sixth of the same space
by each of the following countries: Austria for 13 reports; Great Britain
for 39; Switzerland for 13; and Japan for 16 reports in two volumes.
The reports of France cover nearly 700 pages. Of more modest dimensions
are, in order of size, the reports from Belgium, Russia (including Finland),

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   * A new word *IMUK* has been coined and is used in German writings
as an abbreviation for these last three words.