

## THE SIXTEENTH REGULAR MEETING OF THE SOUTHWESTERN SECTION

The sixteenth regular meeting of the Southwestern Section of this Society was held at the University of Missouri on Saturday, December 1, 1923. The total attendance was thirty-one, including the following eighteen members:

E. F. Allen, Ashton, Blumberg, Brenke, Chittenden, Decherd, Dunkel, E. R. Hedrick, Ingold, Lefschetz, J. V. McKelvey, U. G. Mitchell, Rider, Eugene Stephens, Stouffer, J. S. Turner, R. A. Wells, Westfall.

Professor Hedrick occupied the chair, being relieved during the sessions by Professors Ashton and Brenke. The morning session was devoted to the reading of the papers listed below.

During the afternoon session Professor Henry Blumberg gave a special lecture by invitation of the program committee on *Properties of unrestricted functions*. This lecture will appear in full in an early issue of this BULLETIN.

It was voted to hold the next meeting of the Southwestern Section at Iowa State College, at Ames, Iowa. The following program committee was elected: Professors E. R. Smith (chairman), R. L. Moore, and E. B. Stouffer (secretary).

The titles and abstracts of the papers read are given below. Professor Dickson's paper was read by Professor Hedrick. The papers of Mr. Michal and Professor Meacham were read by title.

1. Professor L. E. Dickson: *Integral solutions of the equation  $x^2 - my^2 = zw$ .*

This paper appeared in the December number of this BULLETIN.

2. Professors E. R. Hedrick and Louis Ingold: *Extensions of Beltrami's equations.*

The equations of Beltrami, which are sometimes used to define a complex function on a two-dimensional surface,

occur in connection with other two-dimensional problems. In this paper these problems and the corresponding Beltrami equations are extended to higher dimensions, in a manner consistent with the paper by the same authors presented to the Society at the Summer Meeting.

3. Professor Louis Ingold: *A symbolic treatment of the geometry of hyperspace.*

In previous papers the author has given a vector interpretation of Maschke's symbols and symbolic differential parameters, with applications to the properties of tangent varieties to arbitrary spaces characterized by a quadratic differential form. In this paper the symbolic method with the vector interpretation is used to study the normal and curvature properties of such spaces.

In addition to the geometric results, a number of new relations are obtained connecting various differential parameters and invariants.

4. Mr. A. D. Michal: *Integro-differential invariants of one-parameter groups of Fredholm transformations.*

The author finds necessary and sufficient conditions that an analytic functional of a function  $y$  and its derivative be invariant under a given arbitrary continuous one-parameter group of linear functional transformations of Fredholm type of the argument  $y$ . Further, it is shown that if the kernel of the infinitesimal transformation is of a certain type, such invariant functionals always exist. The calculation of the invariants in this latter case is then effected.

The functional is assumed to involve  $y$  and its derivative between the values 0 and 1, and, considered as a functional of these two arguments, may, for a given functional value of the derivative of  $y$ , be assigned arbitrarily as a functional of the other argument.

5. Professor S. Lefschetz: *The Kronecker-Poincaré index for certain manifolds.*

In this paper it is shown that the Kronecker-Poincaré index of two manifolds representing in function  $S_4$  the zeros of two functions of two complex variables is equal to their number of intersections. The theorem and its generalization give rise to important and varied applications in algebraic geometry.

6. Professor J. S. Turner: *Rational triangles in which one angle is a rational multiple of another.*

In this paper, a simple method is described for finding the relation between the sides of a triangle  $ABC$  when  $A = nB$ , where  $n$  is rational. The relation is computed, and the complete integral solution (which may also be interpreted as the complete rational solution) is given, when  $n = 2, 3, 4, 5$  and  $3/2$ .

7. Dr. E. F. Allen: *The jacobian of a contact transformation.*

If  $x_1 = X(x, y, p)$ ,  $y_1 = Y(x, y, p)$ ,  $p_1 = P(x, y, p)$  is a contact transformation, it satisfies the Pfaff differential equation  $dy_1 - p_1 dx_1 = \rho(dy - p dx)$ , where  $\rho$  is in general a function of  $x, y$  and  $p$ . It is shown that the jacobian of this transformation is equal to  $\rho^2$ . This is generalized to a space of  $n + 1$  dimensions. It is also shown that the surface  $\rho = 0$  in  $x, y, p$  space is transformed into a curve in  $x_1, y_1, p_1$  space. This is also generalized to a space of  $n + 1$  dimensions.

8. Professor E. B. Stouffer: *On polynomials expressed as determinants with linear elements.*

Dickson has determined the types of general homogeneous polynomials which can be expressed as determinants with linear elements. In the present paper the author obtains Dickson's main results by entirely different means, use being made of the independence of sums of principal minors of determinants.

The method here used gives a simple proof of the well known theorem: *A sufficiently general cubic surface can be generated by three projective bundles of planes.*

9. Professor E. D. Meacham: *Surfaces whose osculating ruled surfaces belong to linear complexes.*

The necessary and sufficient conditions that a ruled surface belong to a linear complex have been given by Wilczynski. Making use of these conditions, the author of this paper investigates those non-ruled surfaces whose osculating ruled surfaces are contained in such linear complexes.

Certain properties of these surfaces and of the linear complexes involved are established, and a geometric construction of the surfaces is given.

10. Professor E. D. Meacham: *Notes on the geometric characterization of the two lines common to four linear complexes associated with a point of a non-ruled surface.*

The author proves the following theorem: At a point  $P$  on a non-ruled surface  $S$ , the two lines common to the four fundamental complexes are the diagonals of a skew quadrilateral whose sides are the four flecnode tangents and whose vertices are the focal points on these sides.

11. Professor E. W. Chittenden: *Nuclear and hypernuclear points in the theory of abstract sets.*

A point  $p$  is a nuclear point of a set  $E$  of power  $\mu$  if every neighborhood of  $p$  contains a subset of  $E$  of power  $\mu$ ; hypernuclear if there exists for every neighborhood  $V$  of  $p$  a subset of  $E$  which is *interior* to  $V$ . The paper states in terms of these concepts necessary and sufficient conditions that a compact set in a space  $V$  be perfectly compact, and that a self-compact set possess the property of Borel-Lebesgue. The second theorem generalizes theorems of Fréchet and Kuratowski-Sierpinski, and is of special interest because it is shown by an example to be independent of the closure of derived sets.

E. B. STOFFER,  
*Secretary of the Section.*