SHORTER NOTICES

The Making of Index Numbers. By Irving Fisher. Boston and New York, Houghton Mifflin Company, 1922. xxxi and 526 pages.

This book is an elaborate inductive study of price index numbers, the prime object being the discovery of the best, or at least the most accurate index number. To begin with, six fundamental types are considered, the arithmetic, geometric, harmonic, median, mode and aggregative. Each of these may be weighted in different ways, thus giving rise to 24 non-identical formulas. According to Fisher an index number should satisfy two chief tests. Since an index number implies two dates, one of which is the base year, the interchange of years yields a second formula called the *time antithesis* of the first. If P_{01} represents the first index number, the time antithesis is $1/P_{10}$, and the first test or *time reversal* test is $P_{01} \cdot P_{10} = 1$.

The second test, original with Professor Fisher, is called the *factor reversal* test, and assumes that index formulas should give consistent results if applied to prices and to quantities. That is

$$P_{01} \cdot Q_{01} = V_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_0}$$

where p_0, p_1 represent prices and q_1, q_0 quantities in the years 0 and 1 respectively. Q_{01} is obtained from P_{01} by interchanging the p's and q's. V_{01} is called the value index. $V_{01} \div Q_{01}$ is called the *factor antithesis* of P_{01}.

Each time antithesis and each factor antithesis is an index number, and after dropping out identicals, 22 new formulas are added to the list making now a total of 46. But only four of these satisfy the first test and none the second. Taking the geometric mean of a pair of time antitheses or of a pair of factor antitheses gives an index number which conforms to the first or to the second test respectively. Performing both operations gives a number which satisfies both tests. By these means the number of formulas is increased to 96 and forms the main series of index numbers discussed in the book. Certain other supplementary formulas giving slight variations to those in the main series are derived by a process called "crossing the weights" and bring the total to 134.

These 134 index numbers now go through a sifting process to find the best one. First all unweighted formulas and all formulas based upon the median or the mode are ruled out because of freakishness

or lack of sensibility. Others are dropped because of more or less bias in one direction, leaving 47 fairly good index numbers. In testing the formulas, the prices and quantities of 36 commodities in the six rather extraordinary years from 1913 to 1918 are used. A surprising thing is the close agreement of this group of index numbers, thus giving the author one of his answers to the argument that the index number to be used should depend upon the purpose for which it is to be employed. Further sifting yields 13 index numbers all satisfying the two tests. Of these the formula

$$\sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \cdot \frac{\sum p_1 q_1}{\sum p_0 q_1}}$$

"is at least equal in accuracy and is probably slightly superior in accuracy to any of the others". This index number is called by Fisher the "ideal" index number, but will probably go into the literature as *Fisher*'s index number, although he insists that the names Walsh and Pigou should be used also.

Eight other numbers are given honorable mention. One in particular, on account of its rapidity of calculation, accuracy and simplicity

$$\frac{\sum (q_0 + q_1) p_1}{\sum (q_0 + q_1) p_0}$$

is really to be preferred to the ideal except when the utmost accuracy is desired, notwithstandig the fact that it is not included in the group of 13 satisfying both tests.

This brief summary gives but a poor idea of the elaborate detail of the book, written as it is for the general reader as well as the specialist. There are worked out examples, tables of comparisons, graphical explanations, notes, appendices for mathematical details until no point is left which is not thoroughly discussed. Chapter 15 on the speed of calculation is a fair example of the painstaking work throughout the book. In this chapter the index numbers for prices and quantities of the above mentioned 36 commodities for the years 1914–1918 were computed, timed and ranked for each of the 134 formulas. The times varied from one hour to 64.5 hours. The ideal formula ranked 29 and took 14.3 hours to calculate. The second of the formulas mentioned above ranked 16 and took 9.6 hours.

We cannot say that Professor Fisher has actually proved that his ideal index number is really the best one, if a best one exists. It seems to be one among several good ones. But the book does prove that some of the index numbers in use are bad ones.

A. R. CRATHORNE