

SHORTER NOTICES

Vorlesungen über algebraische Geometrie. Geometrie auf einer Kurve, Riemannsche Flächen, Abelsche Integrale. By Francesco Severi, translated by Eugen Löffler, with an introduction by A. Brill. Leipzig, B. G. Teubner, 1921. xvi + 408 pp.

There is no fault to find with Dr. Löffler's excellent work of translation, but one regrets to be deprived of the author's clear and limpid Tuscan. We must derive what consolation we may from the hope that what has been lost in artistic quality will be offset by reaching a wider public.

And this question of reaching a wider public is today a crucial one for algebraic geometry, and not for this branch of mathematics alone, be it said by the way. Grafted on function theory on the one hand and on pure geometry on the other, algebraic geometry has reached today, notably in Italy, a very considerable development. Few chapters of mathematics have greater artistic quality, few have evinced quicker growth and cry more loudly for systematic, careful, didactic exposition. This introductory work by one of the keenest and most elegant geometers of our day should be assured a royal welcome.

The *Vorlesungen* are composed of two distinct parts—a very complete exposition of the geometry on algebraic curves, indeed so complete that few questions of interest to geometers are left untouched, and a voluminous supplement. The first is a translation with some amplification of the author's 1908 lithographed lectures. Severi makes the whole theory rest squarely upon linear systems of curves in the plane—an excellent introduction to the more advanced parts of the theory. The author exhibits early his intense geometric bent, his desire to avoid analytic methods. Thus the reduction of singularities is treated without the Puiseux expansions. One misses somehow the finality which comes with these expansions, but the geometer will have his way. . . .

From linear systems of curves to linear series on a given curve is but a step. This is followed by operations on linear series, the extension of which to curves on algebraic surfaces has proved to be such a formidable tool in the hands of Italian geometers. Then come in order the extension to curves in any space and the introduction of the genus and the canonical series by the intrinsic method of Enriques (method of the Jacobian series). To the Riemann-Roche theorem we are led via the C. A. Scott proof of Nöther's fundamental theorem.

A particularly interesting chapter is the treatment of correspondences between two curves or between the points of a given curve. Severi's own work in this direction is of importance second only to that of Hurwitz. Of course his methods, largely geometric, could yield but little on singular correspondences, in the study of which transcendental methods are indispensable. However, the author's treatment has thrown much light on the subject and the connection which he established with an important class of surfaces must have had much to do with one of his main achievements—the discovery of the base for curves on an algebraic surface and its connection with Picard's transcendental theory.

The first part ends with the transcendental theory along the line of Picard's *Traité d'Analyse*, with some asides on real curves and questions of reducibility of abelian integrals, in which direction the author has also made important contributions.

We now come to the Anhang. The volume was ready in August, 1914. In the years that followed, the author, part of the time in active military service, was pursuing investigations on algebraic curves and these with some minor corrections form the subject-matter of the Anhang, which occupies the latter fourth of the work. In form it differs considerably from the rest and in ordinary times we should have expressed the wish to have it more completely merged with the rest, but no doubt this would have greatly delayed publication, and under the circumstances we feel thankful to have it as it is.

The Anhang is certainly the most interesting part of the work for the student of this subject, the one which he will want to read first. Greater preparation is required to read it than for the rest. The material, as interesting as it is new, consists primarily in a series of contributions to our very meager knowledge on families of algebraic curves with singularities assigned in type but not in position. A typical as well as interesting result is this: The manifold of all irreducible curves of given order and genus is itself irreducible. His results along this line lead the author to the first algebraic-geometric proof ever given of Riemann's existence theorem, and also to a new attack on the classification of curves in any space.

May this suffice to whet the reader's appetite and increase his desire to read Severi. He will not repent.

S. LEFSCHETZ.

Theoretical Mechanics. An Introductory Treatise on the Principles of Dynamics with Applications and Numerous Examples. By A. E. H. Love. Third Edition. Cambridge, University Press, 1921. xv + 310 pp.

The first edition of Love's *Mechanics* was published in 1897 and was reviewed in this *BULLETIN* in April, 1898. The second edition, published in 1906, differed from the first in that the material had been rearranged and rewritten, although the general content remained unaltered. The present edition is practically the same as the second, the only additions being a note on *The moment of the kinetic reaction of a particle about a moving axis*, a paragraph on *Force of simple harmonic type*, and a paragraph on *Effect of damping on forced oscillation*. The number of miscellaneous examples has been reduced, although the collection is still sufficiently large to satisfy the demands of the most ardent problems enthusiast.

As the first and second editions have been familiar to students of mechanics for so many years, no further comment seems necessary.

PETER FIELD.

Zur Theorie der Triaden, von Almar Naess. Kristiania, Grondahl & Sons, 1921. 136 pp.

This paper of 136 pages forms Nr. 6, Serie 1, of *NORSK MATEMATISK*