

*Oeuvres Complètes de Cristiaan Huygens*, publiées par la Société Hollandaise des Sciences. Tome XIV. *Calcul des Probabilités. Travaux de Mathématiques Pures, 1655-1666*. La Haye, Martinus Nijhoff, 1920. 524 pp.

Approximately the first one-third (pp. 1-179) of this attractive volume is devoted to contributions to the theory of probability (1656-1657), with appendices most of which are of later date. The next division (pp. 181-407) presents contributions to various subjects (1655-1659) including the theory of numbers, rectification of the parabola, quadrature of conoids, volumes of solids of revolution, centers of gravity, properties of the cycloid, and theory of evolutes. The third division (pp. 409-427) gives contributions to the commentaries of von Schooten on the *Geometria* of Descartes, editions of 1649 and 1659. The fourth division (pp. 429-524) like the second relates to a wide range of subjects. This division (1661-1666) presents contributions to the theory of logarithms including the logarithmic curve with application to the determination of altitude by means of the barometer, further work on solids of revolution, rules for finding the tangent to an algebraic curve, and the theory of cubics. Each division is preceded by a valuable historical introduction.

Although the first steps in the theory of probability were taken by Pascal and Fermat in their correspondence as early as 1654 about questions proposed by the Chevalier de Méré, the work of Huygens constitutes the first treatise on the subject. He starts from the fundamental assumption that there exists for any equitable game a determinate numerical value for the probability that a player will win or lose. He develops a few elementary propositions of which he makes frequent application. Proposition III asserts that if a player has " $p$  chances of gaining  $a$  and  $q$  chances of gaining  $b$ , his expectation is  $(pa + qb)/(p + q)$ ." In present day usage, we should probably avoid the use of the word "chances" in the sense in which it is here used. The thought could be expressed by saying that  $(pa + qb)/(p + q)$  is the expectation of a trial when in a total of  $p + q$  equally likely trials, there are  $p$  ways of gaining an amount  $a$  and  $q$  ways of gaining  $b$ . This proposition affects much that follows it.

The entire treatment of probability contains fourteen propositions, five problems, and nine appendices. The first two propositions are special cases of proposition III just

stated. The propositions IV to IX are concerned with the problem of points with two or three players. The propositions X to XIV deal with the throwing of dice. The nine appendices treat various applications many of which came up in correspondence. The last one, which carries the date 1688, is the problem of the chances of three players at piquet. This problem was reduced by Huygens to the sum of an infinite series. This appendix was written much later than the treatise.

Apart from the division on probability, the volume is devoted mainly to geometry. However, there are about seventeen pages of text given to number theory. This work on number theory relates mainly to the solution of the equation of Pell, and closely related problems. The contributions to geometry vary in character from material as elementary as a new demonstration of the Pythagorean proposition to the determination of surfaces of parabolic and hyperbolic conoids, the tangents to algebraic curves, and the theory of evolutes. Considerable space is given to properties of the cycloid. In this work, Huygens was probably accepting the challenge put up by the letter of Pascal (under an assumed name) addressed "to all geometers of the universe" in which he proposed problems about a certain semisegment of the cycloid calling for the area, the center of gravity, volume of solids generated by revolution about certain lines, and the centers of gravity of these solids. Among the geometric contributions, we should perhaps mention especially that this volume gives properties of the logarithmic curve, and the quadrature of the hyperbola by the use of logarithms.

The division on probability is given both in Dutch and in a French translation. The text of the other divisions is, with a few exceptions, in Latin. The valuable footnotes and introductions to divisions are in French. The volume contains a table of contents, a name index, and a subject index that are very useful. When we consider the preparation of the introductions to the various divisions of the work, the large number of references, and the many footnote explanations and comments found throughout the work, it seems to the reviewer that this volume represents practically a model piece of work from the editorial standpoint as well as a contribution to the history of Huygens's mathematical work.

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